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On peristaltic motion of pseudoplastic fluid in a curved channel with heat/mass transfer and wall properties

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ABSTRACT

This work addresses the combined effect of wall properties and heat/mass transfer on the peristaltic motion of pseudoplastic (shear-thinning/shear-thickening) fluid in a curved channel. The mathematical model is simplified through the assumption of long wavelength of the peristaltic wave compared to the mean half-width of the channel. Series solutions for stream function, temperature and concentration of species are derived. In contrast to the case of planar channel, the profiles are not symmetric about the central line of the curved channel. The size of the trapped bolus is different in the upper and lower halves of the curved channel. Moreover the number of circulations increase/decrease in the upper/lower half of the channel when the case of planar channel is approached.

1. Introduction

Peristalsis is a form of fluid transport that occurs due to the propagation of sinusoidal waves across the walls of channel. This phenomenon widely occurs in several industrial and biomedical applications including swallowing of food through esophagus, transport of toxic liquid in nuclear industry, chyme motion in the gastrointestinal tract, blood circulation in small blood vessels, sanitary fluid transport of corrosive fluids etc. Many modern mechanical devices are designed through peristaltic pumping for fluid transport without internal moving parts. Peristaltic pumping has gained significant attention due to its relevance in ureteral functions. The mathematical models introduced by Shapiro [1], Fung [2] and Shapiro et al. [3] described the biologically and medically important phenomenon of "reflux". Vesicoureteral reflux (VUR) is the backward flow of urine from the bladder into the kidneys. It allows bacteria, which may be present in the urine inside the bladder, to reach the kidneys. This leads to the kidney infection, scarring and damage. The dynamics of ureteral muscles through peristalsis have been explained by Fung [4]. Mishra et al. [5] presented a mathematical model to describe the peristaltic motion of blood under the influence of magnetic field. In view of such applications, peristaltic flow of both viscous and non-Newtonian fluids has been significantly discussed in recent years. The heat transfer analysis on the peristaltic flow in a porous annulus has been investigated by Vajravelu et al. [6]. Srinivas and Kothandapani [7] discussed the peristaltic transport in an asymmetric channel with heat transfer. Mekheimer and elmaboud [8] examined the influence of heat transfer and magnetic field on the peristaltic transport of Newtonian fluid in a vertical annulus. Hayat et al. [9] studied the effect of heat transfer on the MHD peristaltic flow of Newtonian fluid in a porous space.

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Srinivas et al. [10] examined the influence of mixed convective heat and mass transfer effects on peristalsis in an asymmetric channel. Srinivas and Muthuraj [11] studied the effects of chemical reaction and space porosity on MHD mixed convective peristaltic flow of Newtonian fluid in a vertical asymmetric channel. Abd elmaboud [12] analyzed the effects of induced magnetic field on peristaltic flow in an annulus. Tripathi [13] analyzed the peristaltic transport of viscoelastic fluid in a channel. Peristaltic flow of en electrically conducting Carraeu fluid with heat and mass transfer is addressed by Vajravelu et al. [14]. Husseny et al. [15] described the flow separation of peristaltic transport between two coaxial tubes by considering Maxwell fluid. Influences of variable viscosity and thermal conductivity on the peristaltic flow in vertical asymmetric channel have been examined by Mekheimer et al. [16]. Exact analytical solution for peristaltic flow of nanofluid in asymmetric channel with slip conditions have been derived by Aly and Ebaid [17]. Peristaltic flow of carreau fluid with convective boundary condition is discussed by Hayat et al. [18]. Peristaltic flow of Newtonian fluid with viscosity dependent on both radial and axial components was described by Lachiheb [19]. Ebaid [20] derived an exact solution for peristaltic flow of Jeffrey nanofluid. He compared his results with the existing results by the homotopy perturbation method.

Consideration of wall properties such as wall stiffness, wall rigidity, mass per unit area of the channel walls, wall tension etc. is very important in peristalsis. In particular the increased intensity of such effects can significantly influence the blood pressure in human body. Peristaltic motion in a channel with complaint walls has been discussed previously. Investigation of heat transfer in the peristaltic flow with complaint walls was presented by Radhakrishnamacharya and Srinivasulu [21]. Muthu et al. [22] emphasised the influence of wall compliance on the peristaltic flow of micropolar fluid in circular cylindrical tubes. Hayat et al. [23] addressed the peristaltic flow of Johnson-Segalman fluid in a channel with complaint walls. Srinivas and Kothandapani [24] studied the heat and mass transfer effects on MHD peristaltic flow of Newtonian fluid in a porous channel with compliant walls. Mustafa et al. [25] discussed, both analytically and numerically, the peristaltic flow of nanofluid with wall properties. Hina et al. [26] investigated the impact of chemical reaction on the peristaltic motion with complaint walls. The effects slip boundary condition on the peristaltic flow of nanofluid with wall properties were described by Mustafa et al. [27]. Riaz et al. [28] investigated the peristaltic motion of Prandtl fluid in rectangular duct with wall properties. Recently, peristaltic flow of Burgers' fluid in a complaint walls channel was investigated by Javed et al. [29]. Peristaltic flow with complaint walls and Hall current was studied by Gad [30].

All the above mentioned studies deal with the peristaltic flow analysis through planar channel. Since most of the arteries and glandular ducts are curved therefore it seems worthwhile to consider the peristaltic flow analysis in curved channels. However such flow analysis is scarcely presented in the literature. This is because the consideration of curvilinear coordinates in the conservation equations lead to much complicated non-linear differential equations. Sato et al. [31] firstly described the peristaltic flow in a curved channel. Ali et al. [32] extended the study of Sato et al. [31] in a curved channel adopting wave frame formulation. In another paper, Ali et al. [33] described the peristaltic motion of third grade fluid in curved channel. Peristaltic flow in curved channel with wall properties was firstly discussed by Hayat et al. [34]. This work was extended by Hayat et al. [35] for third grade fluid with combined heat and mass transfer effects. Influence of wall properties on the peristaltic motion of third grade fluid has been described by Hina et al. [36]. Very recently, Hina et al. [37] investigated the peristaltic flow of pseudoplastic fluid in curved channel with wall properties and slip conditions.

The purpose of current study is to extend the problem of Hina et al. [37] for heat and mass transfer effects. Heat and mass transfer effects in peristalsis are quite significant especially in oxygenation, hemodialysis, hemodialysis etc. The analytic solutions for velocity, temperature and concentration distributions are presented and discussed by keeping the specific situation of micro-circulation in view. Heat transfer coefficient at the channel walls is evaluated and discussed.

2. Mathematical formulation

We consider the peristaltic flow of pseudoplastic fluid in a curved channel of thickness $2d_1$ coiled in a circle of radius R^* centred at O. The axial direction of the flow is x and the radial direction is r. The components of velocity in the axial and radial directions are u and v respectively. The propagated waves are of the sinusoidal shape which may be defined by

$$r = \pm\eta(x,t) = \pm\left[d_1 + a\sin\frac{2\pi}{\lambda}(x - ct)\right], \tag{1}$$

in which c is the wave speed and a and λ are the wave amplitude and wavelength respectively. The governing equations of the problem are as follows:

$$\frac{\partial v}{\partial r} + \frac{R^*}{r+R^*}\frac{\partial u}{\partial x} + \frac{v}{r+R^*} = 0, \tag{2}$$

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial v}{\partial x} - \frac{u^2}{r+R^*}\right] = -\frac{\partial p}{\partial r} + \frac{1}{r+R^*}\frac{\partial}{\partial r}\{(r+R^*)S_{rr}\} + \frac{R^*}{r+R^*}\frac{\partial S_{xr}}{\partial x} - \frac{S_{xx}}{r+R^*}, \tag{3}$$

$$\rho\left[\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial u}{\partial x} + \frac{uv}{r+R^*}\right] = \frac{1}{(r+R^*)^2}\frac{\partial}{\partial r}\{(r+R^*)^2 S_{rx}\} + \frac{R^*}{r+R^*}\frac{\partial S_{xx}}{\partial x} - \frac{R^*}{r+R^*}\frac{\partial p}{\partial x}, \tag{4}$$