Applied Mathematics and Computation 261 (2015) 60–67

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Efficient and stable generation of higher-order pseudospectral integration matrices

Xiaojun Tang*

School of Aeronautics, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China

ARTICLE INFO

Keywords:
Pseudospectral integration matrices
Chebyshev polynomials
Pseudospectral methods
Integral equations
Integro-differential equations

ABSTRACT

The main purpose of this work is to provide new higher-order pseudospectral integration matrices (HPIMs) for the Chebyshev-type points, and present an exact, efficient, and stable approach for computing the HPIMs. The essential idea is to reduce the computation of HPIMs to that of higher-order Chebyshev integration matrices (HCIMs), and take a very simple and recursive way to compute the HCIMs efficiently and stably. Extensive numerical results show that the new approach for computing the HPIMs has better stability than that of the recently derived Elgindy's approach for large number of Chebyshev-type points.

© 2015 Elsevier Inc. All rights reserved.

1. Introduction

A variety of phenomena can be modeled as ordinary/partial differential equations. Pseudospectral methods are widely used in the numerical solution of these equations, and can be generally grouped into two major categories: differential and integral. In differential pseudospectral methods, the unknown solution is approximated using Lagrange interpolating polynomials and the differential equation is directly discretized using collocation at a specified set of points via pseudospectral differentiation matrices (PDMs) [1–3]. In integral pseudospectral methods, the differential equation is first recast as an equivalent integral equation and the latter is then discretized using collocation at the specified points via pseudospectral integration matrices (PIMs) [4–6].

It is well known that PDMs are severely ill-conditioned when the number of collocation points is large, while PIMs are well-conditioned even for large number of collocation points [4,6]. This is basically because numerical differentiation is inherently sensitive, as small perturbations in input can cause large changes in output, while numerical integration is inherently stable [6]. As a result, PIMs have been widely accepted and applied by many authors to solve differential, integral, and integro-differential equations (see, e.g., [4,6,7] and the references therein). In particular, higher-order PIMs (HPIMs) have received increasing attention over the past years. El-Gendi et al. [8] adopted the Cauchy's formula to compute the HPIM for the Chebyshev–Gauss–Lobatto (CGL) points from the corresponding first-order PIM. Subsequently, Elbarbary [4] presented a new approach for computing the same HPIM based on the exact relation between the Chebyshev polynomials and their derivatives. Recently, Elgindy [5] has derived the above HPIM using an explicit formula for the iterated integrals of Chebyshev polynomials.

The motivation of this paper is to provide two new HPIMs for the Chebyshev–Gauss (CG) and flipped Chebyshev–Gauss–Radau (FCGR) points, respectively, and present an exact, efficient, and stable approach for computing the HPIMs. The essential idea is to reduce the computation of HPIMs to that of higher-order Chebyshev integration matrices (HCIMs), and take a very simple and recursive way to compute the HCIMs efficiently and stably from their lower-order counterparts. This, together with the stable

* Tel.: +86 13571893786.
E-mail address: nputxj@nwpu.edu.cn

http://dx.doi.org/10.1016/j.amc.2015.03.090
0096-3003/© 2015 Elsevier Inc. All rights reserved.

evaluation of Chebyshev polynomials and their constant normalization factors, leads to an exact, efficient, and stable scheme to compute the HPIMs even at thousands of Chebyshev-type points.

The rest of this paper is organized as follows. In Section 2 the Chebyshev polynomials are presented for subsequent developments. The definitions and computation of HCIMs are presented in Section 3. In Section 4 the definitions and computation of HPIMs are provided. Numerical results are shown in Section 5. Finally, Section 6 contains some concluding remarks.

2. Chebyshev polynomials

The Chebyshev polynomials of the first kind are orthogonal polynomials on the interval $[-1, +1]$, and satisfy the following orthogonality relation:

$$\int_{-1}^{+1} \omega_{_T}(\tau) T_i(\tau) T_j(\tau)\,\mathrm{d}\tau = \int_{-1}^{+1} \frac{T_i(\tau)T_j(\tau)}{\sqrt{1-\tau^2}}\,\mathrm{d}\tau = \lambda_{T_i}\delta_{ij} \tag{1}$$

where $\omega_{_T}(\tau) = \frac{1}{\sqrt{1-\tau^2}}$ is the Chebyshev weight function, δ_{ij} is the Kronecker delta function, and λ_{T_i} is the normalization factor, given by

$$\lambda_{T_i} = \begin{cases} \pi, & i = 0 \\ \dfrac{\pi}{2}, & i \neq 0 \end{cases} \tag{2}$$

The three-term recursion formula for the Chebyshev polynomials is given by

$$T_0(\tau) = 1, \quad T_1(\tau) = \tau, \tag{3a}$$

$$T_{n+1}(\tau) = 2\tau T_n(\tau) - T_{n-1}(\tau), \quad n = 1, 2, \ldots \tag{3b}$$

Some important properties of the Chebyshev polynomials are given by [2]

$$T_n(\pm 1) = (\pm 1)^n \tag{4a}$$

$$|T_n(\tau)| \leq 1 \tag{4b}$$

$$T_n(-\tau) = (-1)^n T_n(\tau) \tag{4c}$$

$$T_n(\tau) = \begin{cases} 1 & n = 0 \\ n2^n \displaystyle\sum_{m=0}^{\lfloor n/2 \rfloor} \frac{(-1)^m (n-m-1)!}{2^{2m+1} m!(n-2m)!}\tau^{n-2m}, & n = 1, 2, \ldots \end{cases} \tag{4d}$$

$$T_n(\tau) = \begin{cases} 1, & n = 0 \\ \dfrac{1}{2(n+1)} T'_{n+1}(\tau) = \dfrac{1}{4} T'_2(\tau), & n = 1 \\ \dfrac{1}{2(n+1)} T'_{n+1}(\tau) - \dfrac{1}{2(n-1)} T'_{n-1}(\tau), & n = 2, 3, \ldots \end{cases} \tag{4e}$$

where $\lfloor n/2 \rfloor$ denotes the largest integer less than or equal to $n/2$.

3. Higher-order Chebyshev integration matrices

In this section, the definitions and computation of HCIMs for the CG and FCGR points are presented, respectively.

3.1. Definitions of higher-order Chebyshev integration matrices

Definition 1 ([8]). The HCIM of order $\ell \geq 1$ for the CG points of $\{\tau_i \in (-1, +1)\}_{i=1}^{N}$ with $-1 = \tau_0 < \tau_1 < \cdots < \tau_{N+1} = 1$ is defined as

$${}_{-1}^{\tau}\boldsymbol{T}_{kj}^{\ell} \triangleq \underbrace{\int_{-1}^{\tau_k}\int_{-1}^{\sigma_\ell}\cdots\int_{-1}^{\sigma_3}\int_{-1}^{\sigma_2}}_{\ell} T_j(\sigma_1)\,\mathrm{d}\sigma_1\mathrm{d}\sigma_2\cdots\mathrm{d}\sigma_{\ell-1}\mathrm{d}\sigma_\ell \quad (k = 1, 2, \ldots, N+1, \quad j = 0, 1, \ldots) \tag{5}$$

Definition 2 ([8]). The HCIM of order $\ell \geq 1$ for the FCGR points of $\{\hat{\tau}_i \in (-1, +1]\}_{i=1}^{N}$ with $-1 = \hat{\tau}_0 < \hat{\tau}_1 < \cdots < \hat{\tau}_N = 1$ is defined as

$${}_{-1}^{\tau}\hat{\boldsymbol{T}}_{kj}^{\ell} \triangleq \underbrace{\int_{-1}^{\hat{\tau}_k}\int_{-1}^{\sigma_\ell}\cdots\int_{-1}^{\sigma_3}\int_{-1}^{\sigma_2}}_{\ell} T_j(\sigma_1)\,\mathrm{d}\sigma_1\mathrm{d}\sigma_2\cdots\mathrm{d}\sigma_{\ell-1}\mathrm{d}\sigma_\ell \quad (k = 1, 2, \ldots, N, \quad j = 0, 1, \ldots) \tag{6}$$

Download English Version:

https://daneshyari.com/en/article/4626457

Download Persian Version:

https://daneshyari.com/article/4626457

Daneshyari.com