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Finite-time H_∞ control for a class of discrete-time switched singular time-delay systems subject to actuator saturation ☆

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ABSTRACT

This paper concerns with the finite-time H_∞ control problem for a class of discrete-time switched singular time-delay with actuator saturation. Not only linear matrix inequality conditions for the systems but also average dwell-time of switching signal is given to guarantee regular, causal and finite-time boundedness for the discrete-time switched singular time-delay system. Moreover, sufficient conditions are presented to ensure the H_∞ disturbance attenuation level, and the design method of H_∞ controller is developed by solving linear matrix inequalities (LMI) optimization problem without any decompositions of system matrices and equivalent transformation. Furthermore, the function in the proof procedure belongs to multiple Lyapunov-like functions whose advantage lies in their flexibility. Finally, numerical examples are employed to verify the effectiveness of the proposed methods and to illustrate the significant improvement on the conservativeness of some reported results in the literature.

1. Introduction

Singular systems can be seen as a generalization of the standard state-space systems, which not only describe the dynamics of systems, but also reveal the algebraic constrains [1]. Compare with state-space models, the study of singular systems is more arduous, since not only finite dynamic modes, but also impulsive modes and non-dynamic modes should be taken into account, and the latter two issues do not arise in the state-space case. On the other hand, switched systems, which belong to a special class of hybrid systems, consist of a family of subsystems described by continuous or discrete-time dynamics, and a switching law that specifies the active subsystems at each instant of time. The applications of switched systems can be found in, for example, traffic control [2], network control [3], chemical processing [4] and switching power converters [5]. More and more engineering applications resort to switching strategy [6–10]. Therefore, in the past few decades, special attention has been given to the development of switched singular systems, since they can naturally represent systems and improve control performance [11–15]. Several basic research topics, such as controllability, observability and stability, have attracted much research attention. Many effective methods have been presented to tackle these three basic problems, such as multiple Lyapunov function approach [16], the piecewise Lyapunov function approach [17,18], the switched Lyapunov function approach [19], and the dwell-time, or average dwell-time scheme [11–16].

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Finite-time stability is a concept dealing with the boundedness of the system during a fixed interval. Compared with the largely known stability concept, called Lyapunov asymptotical stability on which most results in the literature focused, the finite-time stability is a different stability concept. As illuminated in [20,21], a system is said to be finite-time stability if once we fix a time-interval, its state does not exceed some bounds during this time-interval. A finite-time bounded system is not necessarily Lyapunov asymptotic stable and Lyapunov asymptotic stable is also not necessarily finite-time stability. Many valuable results have been obtained for this type of stability [22–25]. In [25], the definition of finite-time stability given in [23] was extended and sufficient conditions for finite-time stability and finite-time stabilization of linear system were given via state feedback by using linear matrix inequality. From then on, based on linear matrix inequality theory, a significant improvement has been made in finite-time stability problem [26–36]. In [26–29], the concept of finite-time boundedness which was an extension of finite-time stability was introduced, and some sufficient conditions for finite-time boundedness and stabilization of continuous-time systems or discrete-time systems were presented. In [30], definition of finite-time stability has been extended to the systems with impulsive effects or singular systems with impulsive effect, respectively, and sufficient conditions with impulsive for finite-time stability and stabilization have been also given in [31–36]. For instance, the finite-time H_∞ control of singular systems with parametric uncertainties and disturbances was talked in [31], and the robust finite-time H_∞ control of singular stochastic systems via static output feedback was solved by Zhang et al. in [32]. In [33], a new concept of finite-time stability of linear time-varying singular systems with impulses at fixed time was solved in terms of matrix inequalities. A useful result concerning the combined problem of finite and infinite pole placement by gain output feedback has been established in [34]. Ailon and Beman [35] discussed an open-loop strategy for finite-time control of a given solvable singular system, which was simple for both computation and implementation. In [36], the concept of finite-time stability for continuous descriptor systems was extended to discrete-time switched descriptor systems and the state feedback controllers were designed to guarantee the discrete-time switched descriptor system uniform finite-time stable. For more information of the finite-time control for switched system, see references in [37–40].

On the other hand, actuator saturation can lead to poor performance of the closed-loop systems and sometimes destabilizes the systems. The analysis and design for systems with actuator saturation have received a lot of attentions [41–48]. The robust control for systems with actuator saturation has been discussed in [41,42], respectively, the definition of domain of attraction in mean square sense was introduced. For singular systems with actuator saturation, readers may refer to [43–48] and references therein. For example, Zuo et al. [45] considered the fault tolerant control for singular systems with actuator saturation and nonlinear perturbation. Ma and Zhang [46] gave the sufficient conditions for the stability of discrete-time singular Markov jump systems with actuator saturation, and gave the estimation of the domain of attraction and the design of state feedback gain matrix via LMI technique.

Up to now, Lyapunov stability analysis for switched singular systems and finite-time stability for systems without time-delay and actuator saturation have been extensively studied by many researchers. Due to the requirements for time-delay, actuator saturation and finite-time behavior of a switched singular system in engineering fields to avoid the presence of unacceptable large value states and inputs, it motivates us to study the finite-time stability and stabilization for switched singular time-delay systems with actuator saturation. In this paper, our results are totally different from those previous results, although some results of finite-time stabilization for switched systems have been investigated and the studies mainly focused on the systems without algebraic constraints, time-delay or input saturations, see the references [16,49,50]. To the best of our knowledge, there has been no result being reported for the FTB H_∞ control for switched discrete-time singular time-delay systems with actuator saturation. Hence it is necessary to address this important problem.

The novelty of our research is that finite-time stability, boundedness and stabilization of a class of switched discrete-time singular time-delay systems with actuator saturation are investigated which are important properties for switched system, but neglected by most previous research. The main contribution of this paper is that sufficient conditions ensuring a class of switched discrete-time singular time-delay systems with actuator saturation regular, causal and finite-time bounded are proposed. Furthermore, the average dwell-time is given to guarantee finite-time boundedness of the switched system. Finally, based on the analysis results, a state feedback controller is designed to ensure the closed-loop system H_∞ finite-time bounded. All results can be formulated into LMI form, which facilitate solving procedure of analysis and H_∞ controller design problem. Several numerical examples are presented in the last to show a less conservative than the results in the literature.

2. Problem formulation

Consider discrete-time singular time-delay systems subject to actuator saturation with the following dynamics:

$$\begin{cases} Ex(k+1) = A_{\sigma(k)}x(k) + A_{d,\sigma(k)}x(k-d) + B_{\sigma(k)}\mathrm{sat}(u(k)) + B_{w,\sigma(k)}w(k), \\ z(k) = C_{\sigma(k)}x(k) + C_{d,\sigma(k)}x(k-d) + D_{\sigma(k)}\mathrm{sat}(u(k)) + D_{w,\sigma(k)}w(k), \\ x(k) = \phi(k), \ \ k_0 - d \le k \le k_0, \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^p$ is the control input vector, $z(k) \in R^q$ is the control output vector and $\mathrm{sat} : R^p \to R^p$ is the standard saturation function defined as follows:

$$\mathrm{sat}(u(k)) = [\mathrm{sat}(u_1(k)), \mathrm{sat}(u_2(k)), \ldots, \mathrm{sat}(u_p(k))]^T,$$