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Rate of convergence of Lupas Kantorovich operators based on Polya distribution

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ABSTRACT

In the present paper, we consider the Kantorovich modification of Lupas operators based on Polya distribution. We estimate the rate of convergence for absolutely continuous functions having a derivative coinciding a.e. with a function of bounded variation.

1. Introduction

Let α be a non-negative parameter which may depend on n. Stancu [16] defined a sequence of positive linear operators as follows:

$$P_n^{(\alpha)}(f;x)=\sum_{k=0}^{n} f\left(\frac{k}{n}\right)p_{n,k}^{(\alpha)}(x), \tag{1.1}$$

where $p_{n,k}^{(\alpha)}(x)=\sum_{k=0}^{n}\binom{n}{k}\frac{x^{[k,-\alpha]}(1-x)^{[n-k,-\alpha]}}{1^{[n,-\alpha]}}f(\frac{k}{n})$ and $t^{[n,h]}=t(t-h)\ldots(t-\overline{n-1}h)$. We observe that for $\alpha=0$, these operators include the classical Bernstein polynomials. For the special case $\alpha=\frac{1}{n}$, Lupas and Lupas [12] obtained an explicit representation of (1.1) given by

$$P_n^{(1/n)}(f;x)=\sum_{k=0}^{n} f\left(\frac{k}{n}\right)p_{n,k}^{(1/n)}(x), \tag{1.2}$$

where $f\in C(I)$, with $I=[0,1]$, $p_{n,k}^{(1/n)}(x)=\frac{2(n!)}{(2n)!}\binom{n}{k}(nx)_k(n-nx)_{n-k}$, $(n)_k$ being the rising factorial given by $(n)_k=n(n+1)\ldots(n+k-1)$. Recently, Miclaus [14] studied some approximation properties of these operators.

Gupta and Rassias [7] introduced the Durrmeyer variant of the operators (1.2) and gave some local and global approximation results. These recent works motivated us to study the Kantorovich modification of the operators (1.2). Therefore in [3], to approximate Lebesgue integrable functions, we introduced the following integral modification of the operators (1.2):

$$D_n^{*(1/n)}(f;x)=(n+1)\sum_{k=0}^{n} p_{n,k}^{(1/n)}(x)\int_{\frac{k}{n+1}}^{\frac{k+1}{n+1}} f(t)\,dt, \quad f\in C(I) \tag{1.3}$$

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and investigated the local and global approximation properties. Furthermore, the authors also considered the bivariate form of these operators and discussed the degree of approximation, using the second order Ditzian–Totik modulus of smoothness and the corresponding K-functional.

In the present paper we estimate the rate of convergence of the operators $D_n^{*(1/n)}(f;x)$ for absolutely continuous functions having a derivative coinciding a.e. with a function of bounded variation on I. We notice that the class of functions considered here is much more general than the class of continuously differentiable functions. As it is well known, if f is bounded on [0, 1] and x is a point of discontinuity of the first kind, then $\lim_{n\to\infty} L_n(f;x) = \frac{1}{2}(f(x+)+f(x-))$ with L_n being a linear positive operator (see, [9]). If f is a function of bounded variation on [0, 1] then the above equality holds for all x in (0, 1).

The rate of approximation for functions with derivatives of bounded variation is an interesting topic. In an important work due to Bojanic and Cheng [5], the authors estimated the rate of convergence of the Bernstein polynomials for an absolutely continuous function whose derivative is of bounded variation. In [4], the authors estimated rate of approximation for functions with derivatives of bounded variation and they gave some applications for some important operators. The rate of convergence of summation integral type operators is investigated in [8]. In [11], the convergence rate was estimated for Kantorovich-type operators. The detailed knowledge about this area can be found in ([6], pp. 213–244 and 301–308).

In the recent years, several researchers have made significant contributions in this direction. We refer the reader to some of the related papers (cf. [1],[2],[10], [13] and [15], etc.).

Throughout this paper, $DBV(I)$ denotes the class of all absolutely continuous functions f defined on I, having on I a derivative f' coinciding a.e. with a function of bounded variation on I. We notice that the functions $f \in DBV(I)$ possess a representation

$$f(x) = \int_0^x g(t)\,dt + f(0)$$

where $g \in BV(I)$, i.e., g is a function of bounded variation on I.

The operators $D_n^{*(1/n)}(f;x)$ also admit the integral representation

$$D_n^{*(1/n)}(f;x) = \int_0^1 K_n(x,t)\,f(t)\,dt, \tag{1.4}$$

where the kernel $K_n(x,t)$ is given by

$$K_n(x,t) = (n+1)\sum_{k=0}^{n} p_{n,k}^{(1/n)}(x)\chi_{n,k}(t),$$

where $\chi_{n,k}(t)$ is the characteristic function of the interval $[k/(n+1),(k+1)/(n+1)]$ with respect to I.

2. Auxiliary results

In order to prove our main theorem, we need the following results:

Lemma 1 ([3]). *For $e_i = t^i$, $i = 0, 1, 2$ we have*

(i) $D_n^{*(1/n)}(e_0;x) = 1$;
(ii) $D_n^{*(1/n)}(e_1;x) = \frac{2nx+1}{2(n+1)}$;
(iii) $D_n^{*(1/n)}(e_2;x) = \frac{3n^3x^2+9n^2x-3n^2x^2+3nx+n+1}{3(n+1)^3}$.

Remark 1. By simple applications of Lemma 1, we have

$$D_n^{*(1/n)}(t-x;x) = \frac{2nx+1}{2(n+1)} - x = \frac{1-2x}{2(n+1)}$$

and

$$\begin{aligned} D_n^{*(1/n)}((t-x)^2;x) &= D_n^{*(1/n)}(t^2;x) - 2xD_n^{*(1/n)}(t;x) + x^2D_n^{*(1/n)}(1;x) \\ &= \frac{3n^3x^2+9n^2x-3n^2x^2+3nx+n+1}{3(n+1)^3} - 2x\frac{2nx+1}{2(n+1)} + x^2 \\ &= \frac{3x(1-x)(2n^2-n-1)+(n+1)}{3(n+1)^3} = O\left(\frac{1}{n}\right), \quad \text{as } n\to\infty. \end{aligned}$$

Lemma 2. *For $f \in C(I)$, we have $\|D_n^{*(1/n)}(f;x)\| \le \|f\|$, where $\|.\|$ denotes the sup-norm on I.*

Lemma 3. *For $n \in \mathbb{N}$, we have*

$$D_n^{*(1/n)}((t-x)^2;x) \le \frac{2}{(n+1)}\delta_n^2(x),$$

where $\delta_n^2(x) = \phi^2(x) + \frac{1}{(n+1)}$ and $\phi^2(x) = x(1-x)$.