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Stochastic approach for the solution of multi-pantograph differential equation arising in cell-growth model

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ABSTRACT

In this paper, a computational technique is introduced for the solution of the first order multi-pantograph differential equation (MPDE) through some well-known optimization algorithms like sequential quadratic programming (SQP) and Active Set Technique (AST). Furthermore, artificial neural network (ANN) is used for networking of the first order multi-pantograph differential equation in used to provide mathematical model based on unsupervised error for equation. Moreover, mathematical modeling has been performed perfectly through multi-runs for simulation to justify the better convergence of the solutions. Also, two examples are presented to exhibit the aptitude of the method SQP and AST. The comparative study will be made with reported techniques such as variational iteration technique (VIT) [6] and collocation based on Bernstein polynomial method (BCM) [6].

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1. Introduction

The multi-pantograph differential equation (MPDE) is a functional differential equation that was first used by the Tayler and Ockendon for current collection system for an electric locomotive system. Also there exists a lot of work through analytical and approximate techniques in different times and researchers. But every technique has its own limitations for the numeric treatment of MPDE.

The computing approaches used are the local search technique sequential quadratic programming (SQP) and active set technique (AST). The mathematical modeling performed by artificial neural network (ANN), mimicry of the actual neural system.

Functional differential equation with proportional delays is known as pantograph differential equation or generalized pantograph differential equation [1]

$$U'(t) = aU(t) + bU(qt) + cU'(qt) \tag{1}$$

where $q \in (0, 1)$, $U(0) = 1$ and a, b, c are complex [2–5]

The word pantograph originated from a project (collection of current by the pantograph head of an electric locomotive system) by Ockendon and Tayler [1]. Our required equation is MPDE which is functional differential equation with proportional delays

$$U'(t) - \lambda U(t) - \sum_{j=1}^{m} \mu_j(t)U(q_j t) - f(t) = 0 \tag{2}$$

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and $t \geq 0$ with the initial condition

$$U(0) = \gamma \tag{3}$$

Here λ, γ are real constants, m is a natural number, $0 < q_j < 1$ and $\mu_j(t), f(t)$ are analytical functions [6–8].

Pantograph differential equation has significant behavior in different branches of study. The pantograph equations has applications in current collection system [1], cell growth model [9–11], to attain the fusion of light in the spiral galaxies [12], ruin problem in risk theory [13], in quantum theory [14] and also have some industrial applications.

According to this growing interest of pantograph differential equation in different times there exists a number of variety of analytical and numerical studies by a number of researchers. Soft computing approach to the equation or numerical results attained to reduce error of numeric results and for better apprehension. There are some reported methodologies like as Taylor polynomials [15,16], Runge–Kutta methods with a variable step size [17], Taylor method [18], Variational Iteration method [19,20], collocation method using Hermite polynomials and Bessel collocation method [21,22], direct operatorial tau methodology [23], Chebyshev polynomials [24], collocation method based on Bernoulli operational matrix [8] and first Boubaker polynomials [25,26].

The rest of the paper is organized as Section 2 describes the mathematical model with procedural steps for the respective equation, while inside Sections 3, 4, 5 and 6 are describing about the numerical treatment, first order numerical illustrations of MPDE, the statistical analysis and conclusion respectively.

2. Mathematical modeling with procedural steps

Models are always the vital step to approach better understanding and to reveal the practical aspects of object. In the same manner, mathematical modeling is performed for first order MPDE by uses of artificial neural networks (ANN) based on artificial intelligence (AI). Mathematical model of MPDE (2) is constructed with the help of known vitality of feed-forward ANN, in the form of following continuous mapping for the solution $U(t)$ and its first derivative dU/dt respectively. ANN contains 3-steps formulation criteria:

Step 1:

The first step of ANN is to build a system of equation for the solution and 1st derivative of respective solution of Eq. (2). We define log-sigmoid function (LS) as activation function in the system for the solution of problem is defined as:

$$LS = \sum_{i=1}^{L} \frac{1}{1 + e^{-(\omega_i t + \beta_i)}} \tag{4}$$

The LS function keeps derivatives positive at every point and also it is bounded differentiable real valued function [27,28] .

Step 2:

Now neural network of Eq. (2) with LS function (4) can be formulated as:

$$\hat{U}_{LS}(t) = \sum_{i=1}^{L} \left[\frac{\alpha_i}{1 + e^{-(\omega_i t + \beta_i)}} \right] \tag{5}$$

$$\frac{d\hat{U}_{LS}}{dt} = \sum_{i=1}^{L} \frac{\alpha_i \omega_i e^{-(\omega_i t + \beta_i)}}{\left[1 + e^{-(\omega_i t + \beta_i)}\right]^2} \tag{6}$$

where, α_i, ω_i and β_i are optimization weights and L is number of neurons.

Step 3:

A figure of merit or fitness function is developed by defining an unsupervised manner and it is defined by the two mean square errors

$$E = E_1 + E_2 \tag{7}$$

where E_1 is the error function associated with differential Eq. (2) and it is defined as

$$E_1 = \frac{1}{M} \sum_{p=1}^{M} \left[\hat{U_p}' - \lambda \hat{U}_p - \sum_{j=1}^{m} \mu_j(t_p) \hat{U}(q_j t_p) - f(t_p) \right]^2 \tag{8}$$

where $t \geq 0$ and $M = \frac{1}{h}$, $\hat{U_p} = \hat{U}(t_p)$, $t_p = ph$, with step size h.

And E_2 is the error function which associated with initial condition (3)

$$E_2 = (\hat{U_0} - \gamma)^2 \tag{9}$$

where $\hat{U_0} = \hat{U}(0)$.

In order, to perform the numerical experimentation for the MPDE, we are going to use two major platforms i.e. MATLAB (2012a) and Mathematica softwares and to perform operations through the optimization techniques we use **Optimtool** in MATLAB. Its architecture diagram is presented in Fig. 1.

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