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Further results on stability analysis of discrete-time systems with time-varying delays via the use of novel convex combination coefficients

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ABSTRACT

This paper focuses on deriving an improved stability criterion for discrete time-delay systems via a Lyapunov–Krasovskii functional approach that takes advantage of triple summation terms. To this end, novel convex combination coefficients coupled with second-order time-varying delay terms are introduced, and the corresponding reciprocally convex approach is established with the use of the minimal number of slack matrix variables. Finally, three numerical examples are provided to illustrate the effectiveness of the proposed method, with particular regard to robustness and $\mathcal{H}_\infty$ performance.

1. Introduction

Over the past few decades, the need for further research on the stability of discrete-time systems with time-varying delays has been emphasized (see, for example, [1–6] and references therein). Furthermore, in accordance with the literature cited above, considerable attention has been devoted to deriving less conservative stability criteria, particularly based on the following approaches: the free-weighting matrix approach [7–10], the descriptor system approach [11–13], the Jensen inequality approach [14–18], the delay-partitioning approach [20–23], and the convex combination approach [19,27,30–32]. Here, the free-weighting matrix and the delay-partitioning approaches have been well known to be less conservative than others that use over-bounding techniques. However, one of the negative impacts of such approaches is that a significant increase is expected in the number of decision variables used for stability criteria. Thus, establishing an efficient method for reducing both computational burden and conservatism of stability criteria has emerged as a topic of growing interest.

In order to develop such efficient methods, substantial efforts have been invested into improving the Jensen inequality approach, which requires fewer decision variables than other approaches. In the same vein, [24] has provided a method for incorporating the zero equality terms given in [25] into the stability criterion based on a combined Jensen inequality and the free-weighting matrix approach. After that, with a view to obtaining a less conservative stability criterion, [10] has proposed an extended free-weighting matrix approach based on a Lyapunov–Krasovskii functional approach that gives special consideration to triple summation terms. More recently, based on an augmented Lyapunov–Krasovskii functional with triple summation terms, [26] has addressed the great computational complexity of [10] by restricting the use of too many decision variables. However, it should be noted that there have been almost no attempts to reduce computational complexity while considering the following separation process before the Jensen inequality: $\sum_{j=\underline{d}+1}^{\bar{d}}\sum_{i=k-j}^{k-\underline{d}-1}(\bullet) = \sum_{j=\underline{d}+1}^{d(k)}\sum_{i=k-j}^{k-\underline{d}-1}(\bullet) + \sum_{j=d(k)+1}^{\bar{d}}\sum_{i=k-j}^{k-\underline{d}-1}(\bullet)$. For this

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reason, this paper makes every possible effort to derive an improved stability criterion by exploiting novel convex combination coefficients that arise from the separation process.

This paper focuses on addressing the problem of delay-dependent stability analysis for discrete-time systems with time-varying delays, based on a Lyapunov–Krasovskii functional approach that takes advantage of triple summation terms. The main contributions of this paper include the following. First, that despite the requirement of reduced computational burden, the derived stability criterion achieves identical or better performance in comparison with other results relevant to the use of triple summation terms. Second, that novel convex combination coefficients coupled with second-order time-varying delay terms are introduced, and the corresponding reciprocally convex approach, inspired by [27], is established with the use of the minimal number of slack matrix variables. As a result, the proposed method plays an important role in allowing the aforementioned separation process to be carried out before use of the Jensen inequality. Finally, three numerical examples are provided to show the effectiveness of the proposed method, with particular regard to robustness and $\mathcal{H}_\infty$ performance.

Notation: The notations $X \geq Y$ and $X > Y$ mean that $X - Y$ is positive semi-definite and positive definite, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry. For any square matrix $\mathcal{Q}$, $\mathbf{He}[\mathcal{Q}] = \mathcal{Q} + \mathcal{Q}^T$. For any discrete-time function g_k, $\Delta[g_k]$ denotes its forward difference as $\Delta[g_k] = g_{k+1} - g_k$. The Lebesgue space $\mathcal{L}_{2+} = \mathcal{L}_2[0, \infty)$ consists of square-summable functions on $[0, \infty)$.

2. System description and useful properties

Let us consider the following delayed system

$$\begin{cases} x_{k+1} = Ax_k + A_d x_{k-d(k)} \\ x_k = \phi_k, k \in \{-\bar{d}, \ldots, 0\} \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ and $x_{k-d(k)} \in \mathbb{R}^{n_x}$ are the state and the delayed state, respectively. Here, the state delay $d(k)$ is assumed to be an interval time-varying-type integer: $\underline{d} \leq d(k) \leq \bar{d}$, where $\underline{d}$ and $\bar{d}$ are known positive integers. To facilitate the derivation of the main result, we set $d_0 = 0$, $d_1 = \underline{d}$, $d_2 = d(k)$, and $d_3 = \bar{d}$. Further, we define an augmented state ζ_k as $\zeta_k = [x_k^T x_{k-d_1}^T x_{k-d_2}^T x_{k-d_3}^T \, \Delta x_k^T \sum_{i=k-d_1}^{k-d_0-1} x_i^T \sum_{i=k-d_2}^{k-d_1-1} x_i^T \sum_{i=k-d_3}^{k-d_2-1} x_i^T]^T \in \mathbb{R}^{n_\zeta}$ (in which $n_\zeta = 8n_x$), and establish block entry matrices $\mathbf{e}_i$ such that $x_k = \mathbf{e}_0 \zeta_k$, $x_{k-d_1} = \mathbf{e}_1 \zeta_k$, $x_{k-d_2} = \mathbf{e}_2 \zeta_k$, $x_{k-d_3} = \mathbf{e}_3 \zeta_k$, $\Delta x_k = \mathbf{e}_4 \zeta_k$, $\sum_{i=k-d_1}^{k-d_0-1} x_i = \mathbf{e}_5 \zeta_k$, $\sum_{i=k-d_2}^{k-d_1-1} = \mathbf{e}_6 \zeta_k$, and $\sum_{i=k-d_3}^{k-d_2-1} = \mathbf{e}_7 \zeta_k$.

Property 1. *Consider the time-varying parameters of the following form:*

$$\alpha_1 = \frac{\sum_{j=d_1+1}^{d_2} \sum_{i=k-j}^{k-d_1-1}(1)}{\sum_{j=d_1+1}^{d_3} \sum_{i=k-j}^{k-d_1-1}(1)} \geq 0, \quad \alpha_2 = \frac{\sum_{j=d_2+1}^{d_3} \sum_{i=k-j}^{k-d_1-1}(1)}{\sum_{j=d_1+1}^{d_3} \sum_{i=k-j}^{k-d_1-1}(1)} \geq 0. \tag{2}$$

Then, in view of the fact that $\sum_{j=d_1+1}^{d_2} \sum_{i=k-j}^{k-d_1-1}(1) + \sum_{j=d_2+1}^{d_3} \sum_{i=k-j}^{k-d_1-1}(1) = \sum_{j=d_1+1}^{d_3} \sum_{i=k-j}^{k-d_1-1}(1)$, *the parameters* α_i *given from the double-summations with time-varying delay* $d_2 = d(k)$ *have the following property*: $\alpha_1 + \alpha_2 = 1$.

Property 2. *Consider the time-varying parameters of the following form:*

$$\theta_1 = \frac{\sum_{i=k-d_2}^{k-d_1-1}(1)}{\sum_{i=k-d_3}^{k-d_1-1}(1)} \geq 0, \quad \theta_2 = \frac{\sum_{i=k-d_3}^{k-d_2-1}(1)}{\sum_{i=k-d_3}^{k-d_1-1}(1)} \geq 0. \tag{3}$$

Then, from $\sum_{i=k-d_2}^{k-d_1-1}(1) + \sum_{i=k-d_3}^{k-d_2-1}(1) = \sum_{i=k-d_3}^{k-d_1-1}(1)$, *it follows that* $\theta_1 + \theta_2 = 1$.

Lemma 1 ([18,26])**.** *For any positive-definite matrix* $\mathcal{Q}$ *and vector-valued function* χ_i, *the following inequalities hold:*

$$-\sum_{k-d_q}^{k-d_p-1} \left(\chi_i^T \mathcal{Q} \chi_i\right) \leq -\frac{1}{\sum_{k-d_q}^{k-d_p-1}(1)} \left(\sum_{k-d_q}^{k-d_p-1} \chi_i\right)^T \mathcal{Q} \left(\sum_{k-d_q}^{k-d_p-1} \chi_i\right), \tag{4}$$

$$-\sum_{j=d_p+1}^{d_q} \sum_{i=k-j}^{k-d_n-1} \left(\chi_i^T \mathcal{Q} \chi_i\right) \leq -\frac{1}{\sum_{j=d_p+1}^{d_q} \sum_{i=k-j}^{k-d_n-1}(1)} \left(\sum_{j=d_p+1}^{d_q} \sum_{i=k-j}^{k-d_n-1} \chi_i\right)^T \mathcal{Q} \left(\sum_{j=d_p+1}^{d_q} \sum_{i=k-j}^{k-d_n-1} \chi_i\right). \tag{5}$$