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Existence results for an impulsive fractional integro-differential equation with state-dependent delay

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ABSTRACT

In this paper, we have a tendency to implement different fixed point theorem [Banach contraction principle, Krasnoselskii's [18] and Schaefer's [, 18] coupled with solution operator to analyze the existence and uniqueness results for an impulsive fractional integro-differential equations (IFIDE) with state-dependent delay (SDD) in Banach spaces. Finally, cases are offered to demonstrate the concept.

1. Introduction

The concept of semigroups of bounded linear operator is meticulously associated to solving differential and integro-differential equations in Banach spaces. Recently, this concept has been employed to a significant type of non-linear differential equations in Banach spaces. For more points of interest on this concept, we allude the reader to Pazy [1]. The investigation of impulsive functional differential or integro-differential frameworks is signed up with to their application in strengthening techniques and phenomena conditional on short-time perturbations in the course of their progress. The perturbations are conducted separately and their term is insignificant in correlation with the aggregate length of time of the procedures. For additional purposes of enthusiasm on this concept and on its uses, see for example the treatise by Lakshmikantham et al. [2], Stamova [3], Graef et al. [4], Bainov and Covachev [5], Benchohra et al. [6] and the papers [7–16], and the references cited therein.

The concept of fractional differential equations is growing as an essential place of research due to the fact it is better in problems in evaluation with the corresponding concept of traditional differential equations [17–19]. In fact, such designs can be regarded as an effective substitute to the traditional nonlinear differential designs to imitate many complicated procedures. In latest years, as the historical specialized mathematicians predicted, fractional differential equations have been discovered to be a highly effective tool in many areas, such as viscoelasticity, electro-chemistry, control, porous media, and electromagnetic. For fundamental certainties about fractional systems, one can make reference to the books [20–24], and the papers [25–41], and the references cited therein. Fractional equation with delay features happen in several areas such as medical and physical with state-dependent delay or non-constant delay. These days, existence results of mild solutions for such problems became very attractive

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and several researchers working on it. As of late, few number of papers have been released on the fractional order problems with state-dependent delay [42–45,47–52] and references therein.

As of late, dos Santos et al. [48] analyzed the existence of solutions for FIDE with SDD in Banach spaces. Kavitha et al. [47] acknowledged the existence of mild solutions for FIDE with SDD by using appropriate fixed point theorem. In [49,50], the writers offer adequate circumstances for the existence of solutions of fractional functional differential equation with SDD. Lately, Aissani and Benchohra [46] researched the existence of mild solutions on a compact interval for FIDE with SDD in Banach spaces. However, existence results for IFIDE with SDD in $\mathscr{B}_h$ phase space adages have not yet been completely examined.

To take into consideration of fractional systems in the infinite dimensional space, the primary vital step is to determine a recent strategy of the mild solution. Lately, in Wang et al. [19], an appropriate idea of mild solutions was presented and also they deeply discussed the existing *PC*-mild solution defined by several researchers.

Motivated by the effort of the aforementioned papers [18,19,43], the principle motivation behind this paper is to research the existence of mild solutions for an IFIDE with SDD of the model

$$ {}^{C}D_t^{\alpha}x(t) = \mathscr{A}x(t) + f\big(t, x_{\varrho(t,x_t)}, (\mathscr{H}x)(t)\big), \quad \text{a.e. on} \quad \mathscr{J} - \{t_1, t_2, \cdots, t_m\}, \tag{1.1} $$

$$ \Delta x(t_k) = \mathcal{I}_k(x(t_k^-)), \quad k = 1, 2, \cdots, m, \tag{1.2} $$

$$ x(t) = \varsigma(t), \quad \varsigma(t) \in \mathscr{B}_h, \tag{1.3} $$

where $\mathscr{J} = [0, b]$ with $b > 0$ is settled, ${}^{C}D_t^{\alpha}$ is the Caputo fractional derivative of the order $\alpha \in (0, 1)$ with the lower limit zero, $\mathscr{A}$ is a fractional sectorial operator similar to [53] described on a Banach space $\mathbb{X}$ having its norm recognized as $\|\cdot\|_{\mathbb{X}}$, $f : \mathscr{J} \times \mathscr{B}_h \times \mathbb{X} \to \mathbb{X}$, is supplied $\mathbb{X}$-valued functions, $\mathscr{H}$ is described as

$$ (\mathscr{H}x)(t) = \int_0^t e(t, s, x_{\varrho(s,x_s)})ds, $$

where $e : \mathscr{D} \times \mathscr{B}_h \to \mathbb{X}$, $\varrho : \mathscr{J} \times \mathscr{B}_h \to (-\infty, b]$ are apposite functions, and $\mathscr{B}_h$ is a phase space characterized in preliminaries. Here $\mathscr{D} = \{(t, s) \in \mathscr{J} \times \mathscr{J} : 0 \le s \le t \le b\}$. Here $0 = t_0 < t_1 < t_2 < \cdots < t_m < t_{m+1} = b$, $\mathcal{I}_k : \mathbb{X} \to \mathbb{X} (k = 1, 2, \cdots, m)$ are impulsive functions which portray the jump of the solutions at impulse points t_k, and $x(t_k^+) = \lim_{h\to 0} x(t_k + h)$, $x(t_k^-) = \lim_{h\to 0} x(t_k - h)$ are the right and left limits of x at the points t_k separately.

For almost any continuous function x characterized on $(-\infty, b]$ and any $t \ge 0$, we designate by x_t the part of $\mathscr{B}_h$ characterized by $x_t(\theta) = x(t + \theta)$ for $\theta \le 0$. Now $x_t(\cdot)$ speaks to the historical backdrop of the state from every $\theta \in (-\infty, 0]$ likely the current time t.

In contrast to the the current outcomes, this paper has some positive aspects: First of all, we include the operator $\mathscr{H}$ in the non-linear term f and present an appropriate notion of mild solution of the model (1.1)–(1.3). Then, we analyze the existence of mild solutions for IFIDE with SDD of the design (1.1)–(1.3) under different fixed point theorems, and other sorts of hypotheses are more general in contrast to the current investigation; [see the assumptions (H1)–(H3) and (H5)–(H7)], and the results in [16,18,46] might be observed as the special circumstances. And also, we implement $\mathscr{B}_h$ phase space adages to examine the model (1.1)–(1.3).

This paper is composed as takes after. In section 2, we show a few preliminaries and lemmas that are to be utilized subsequently to demonstrate our primary outcomes. In section 3, the existence of mild solutions for the model (1.1)–(1.3) is discussed under different fixed point theorems. Section 4 is saved for a case to delineate the conceptual results.

2. Preliminaries

In this part, we display a few documentations, definitions and preparatory facts from functional analysis, solution operator and fractional calculus theory which will stand for utilized all through this paper.

Let $L(\mathbb{X})$ symbolizes the Banach space of all bounded linear operators from $\mathbb{X}$ into $\mathbb{X}$, having its norm recognized as $\|\cdot\|_{L(\mathbb{X})}$. Moreover, $B_r(x, \mathbb{X})$ symbolizes the closed ball in $\mathbb{X}$ with the middle at x and the distance r.

It needs to be outlined that, once the delay is infinite, then we need to talk about the theoretical phase space $\mathscr{B}_h$ in a beneficial way. In this paper, we deliberate phase spaces $\mathscr{B}_h, \mathscr{B}'_h$ which are same as described in [18]. So, we bypass the details.

If $x : (-\infty, b] \to \mathbb{X}, b > 0$, is continuous on $\mathscr{J}$ and $x_0 \in \mathscr{B}_h$, then for every $t \in \mathscr{J}$ the accompanying conditions hold:

(P_1) x_t is in $\mathscr{B}_h$;
(P_2) $\|x(t)\|_{\mathbb{X}} \le H\|x_t\|_{\mathscr{B}_h}$;
(P_3) $\|x_t\|_{\mathscr{B}_h} \le \mathscr{D}_1(t)\sup\{\|x(s)\| : 0 \le s \le t\} + \mathscr{D}_2(t)\|x_0\|_{\mathscr{B}_h}$, where $H > 0$ is a constant and $\mathscr{D}_1(\cdot) : [0, +\infty) \to [0, +\infty)$ is continuous, $\mathscr{D}_2(\cdot) : [0, +\infty) \to [0, +\infty)$ is locally bounded, and $\mathscr{D}_1, \mathscr{D}_2$ are independent of $x(\cdot)$.
(P_4) The function $t \to \varsigma_t$ is well described and continuous from the set

$$ \mathcal{R}(\varrho^-) = \{\varrho(s, \psi) : (s, \psi) \in [0, b] \times \mathscr{B}_h\}, $$

into $\mathscr{B}_h$ and there is a continuous and bounded function $J^{\varsigma} : \mathcal{R}(\varrho^-) \to (0, \infty)$ to ensure that $\|\varsigma_t\|_{\mathscr{B}_h} \le J^{\varsigma}(t)\|\varsigma\|_{\mathscr{B}_h}$ for every $t \in \mathcal{R}(\varrho^-)$.