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Analysis of a stochastic logistic model with diffusion

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ABSTRACT

Taking both white noise and Lévy jump noise into account, a stochastic logistic model with diffusion is proposed and considered. Under some simple assumptions, the almost complete parameters analysis of the model is carried out. In each case it is shown that the population in each patch is either stable in time average or extinct, depending on the parameters of the model, especially, depending on the intensity of the Lévy jump noise. Some simulation figures are introduced to validate the theoretical results.

1. Introduction

In the natural world, it is a common phenomenon that the individuals of species move from high population densities to low to avoid crowding or congestion, for instance, arctic ground squirrels and vole species [1]. Thus many population models with diffusion have been proposed and analyzed. Hastings [2] investigated the local stability of a positive equilibrium for a deterministic single-species model with diffusion. Beretta and Takeuchi [3] established the sufficient conditions for uniqueness and global stability of the positive equilibrium of their model. Then the results in [3] were improved by Li and Shuai [4] by using the graph-theoretic approach to the construction of Lyapunov functions. Allen [5] considered the following logistic model with diffusion:

$$\frac{dx_i(t)}{dt} = x_i(t)[r_i - a_ix_i(t)] + \sum_{j=1, j\neq i}^{n} D_{ij}x_i(t)[x_j(t) - \alpha_{ij}x_i(t)], \quad i = 1, \dots, n, \tag{1}$$

where $x_i (i = 1, 2, \dots, n)$ is the species x in patch i, $r_i > 0$ stands for the growth rate of species x in patch i, $a_i > 0$ represents the self-inhibition coefficient, $D_{ij} \geq 0$ is the dispersal coefficient of species x from patch j to patch i, $\alpha_{ij} \geq 0$ "corresponds to the boundary conditions of the continuous diffusion case, $\alpha_{ij} = 1$ for Neumann condition, $\alpha_{ij} \neq 1$ for Dirichlet or Robin conditions" [5], $i, j = 1, \dots, n$. Allen [5] (Proposition 1) has shown that if $n = 2$ and $\Lambda := \bar{l}_1\bar{l}_2 - D_{12}D_{21} > 0$, then the unique positive equilibrium $x^* = (\Lambda_1/\Lambda, \Lambda_2/\Lambda)$ of model (1) is globally asymptotically stable, where $\bar{l}_1 = a_1 + D_{12}\alpha_{12}$, $\bar{l}_2 = a_2 + D_{21}\alpha_{21}$, $\Lambda_1 = r_1\bar{l}_2 + r_2D_{12}$, $\Lambda_2 = r_2\bar{l}_1 + r_1D_{21}$. More studies in this direction can be found in [6–9] and references cited therein.

Population growth in the natural world is inherently stochastic because of numerous unpredictable causes [24,25]. Therefore it is important to investigate stochastic population models with diffusion, and to reveal the effects of environmental noises on the properties of the models. By taking the white noise into account, the authors [10] considered the following stochastic logistic

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model with diffusion

$$dx_i(t) = x_i(t)[r_i - a_i x_i(t)]dt + \sum_{j=1, j\neq i}^{n} D_{ij} x_i(t)[x_j(t) - \alpha_{ij} x_i(t)]dt + \sum_{j=1}^{m} \sigma_{ij} x_i(t) dW_j(t), \quad i = 1, \ldots, n \tag{2}$$

where $W_1(t), W_2(t), \ldots, W_m(t)$ are independent standard Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, σ_{ij}^2 is the intensity of the white noise, $i = 1, \ldots, n$, $j = 1, \ldots, m$. Under the conditions that $n = 2$ and $\Lambda > 0$, the authors [10] have obtained the critical value between stability in time average and extinction for the population in each patch of model (2).

Motivated by [10], some interesting topics arise naturally.

(Q1) The authors [10] established the critical value for a 2-patches system, it is interesting is to extend the critical value results of 2-patches system to general n-patches system.

(Q2) The authors [10] used only white noise to describe the environmental perturbations. However, the natural growth of population often suffer sudden random environmental perturbations, such as epidemics, harvesting, earthquakes, hurricanes, etc., which cannot be described by the white noise. Then how to model these sudden random environmental perturbations, and what are the impacts of these sudden random environmental perturbations on the dynamics of population models?

The aim of this paper is to study these problems. Several authors (see e.g., [11–18]) have suggested that one may use Lévy jumps to describe the sudden random environmental perturbations. Thus in this paper, we introduce jumps into model (2) and then obtain the following stochastic model with Lévy noise:

$$\begin{aligned} dx_i(t) &= x_i(t)[r_i - a_i x_i(t)]dt + \sum_{j=1, j\neq i}^{n} D_{ij} x_i(t)[x_j(t) - \alpha_{ij} x_i(t)]dt \\ &\quad + \sum_{j=1}^{m} \sigma_{ij} x_i(t) dW_j(t) + \int_{\mathbb{Z}} \gamma_i(u) x_i(t^-) \tilde{\Upsilon}(dt, du), \quad i = 1, \ldots, n \end{aligned} \tag{3}$$

where $x_i(t^-)$ is the left limit of $x_i(t)$. $\tilde{\Upsilon}(dt, du) = \Upsilon(dt, du) - \theta(du)dt$, Υ is a Poisson counting measure with characteristic measure θ on a measurable subset $\mathbb{Z}$ of $(0, +\infty)$ with $\theta(\mathbb{Z}) < +\infty$. Under some simple assumptions, the almost complete parameters analysis of system (3) is carried out in Section 2. In each case it is shown that the population in each patch is either stable in time average or extinct, depending on the parameters of model (3), especially, depending on the intensity of the Lévy jump noise. In Section 4, we introduce some simulation figures to validate the theoretical results. In Section 5, we give some concluding remarks.

2. Main results

For simplicity, we introduce the following notations.

$$R_+^n = \{\xi \in R^n | \xi_i > 0, i = 1, \ldots, n\}, \quad \langle f(t) \rangle = t^{-1} \int_0^t f(s)ds, \quad f^* = \limsup_{t\to+\infty} f(t), \quad f_* = \liminf_{t\to+\infty} f(t),$$

$$\beta_i = 0.5 \sum_{j=1}^{m} \sigma_{ij}^2 + \int_{\mathbb{Z}} [\gamma_i(u) - \ln(1 + \gamma_i(u))]\theta(du), \quad i = 1, \ldots, n;$$

For a symmetric $n \times n$ matrix $Q = (q_{ij})_{n\times n}$, define $\lambda_{\max}^+(Q) = \sup_{x\in R_+^n, |x|=1} x^T Q x$.

Let $\bar{A} = (b_{ij})_{n\times n}$, where $b_{ii} = a_i + \sum_{j=1, j\neq i}^{n} D_{ij}\alpha_{ij} > 0$, $b_{ij} = -D_{ij} < 0$, $i, j = 1, \ldots, n$, $j \neq i$. $A = \det(\bar{A})$.
A_k is the k-order leading principal minor of A;
$(A_k)_{ij}$ is the complement minor of the element b_{ij} in A_k;
$A_k^{(i)}$ is the k-order determinant by removing the ith row and the ith column of A_k;
A_k^i denotes the k-order determinant by changing the ith column of determinant A_k to $(r_1, \ldots, r_k)^T$;
$\tilde{A}_k^i$ represents the k-order determinant by changing the ith column of A_k to $\left(\beta_1, \ldots, \beta_k\right)^T$;
B_k^{ij} is the k-order determinant by changing the ith column of A_k to $(r_1, r_2, \ldots, r_k)^T$ and the jth column to $\left(\beta_1, \ldots, \beta_k\right)^T$;
$C_k^{(i)}$ is the k-order determinant by removing the ith row and the ith column of C_k, where $C_k = A_k^k$.
$\tilde{C}_k^{(i)}$ is the k-order determinant by removing the ith row and the ith column of $\tilde{C}_k$, where $\tilde{C}_k = \tilde{A}_k^k$.

Throughout this paper, without loss of generality, we assume that

$$\frac{r_1}{\beta_1} > \frac{r_2}{\beta_2} > \cdots > \frac{r_n}{\beta_n},$$

From the viewpoint of biology, this assumption means that the order of persistent ability of the species (from strongest to weakest) is $x_1, x_2, \ldots, x_n$. As a standing hypothesis we assume that Υ and $W_j(t)$ are independent, $j = 1, \ldots, m$. We also assume that