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Periodic orbits of a neuron model with periodic internal decay rate

I. Bula [a,b,*], M.A. Radin [c]

[a] University of Latvia, Zeļļu 8, Rīga LV-1002, Latvia
[b] Institute of Mathematics and Computer Science of University of Latvia, Raiņa bulv. 29, Rīga LV-1048, Latvia
[c] Rochester Institute of Technology, Rochester, New York, USA

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ABSTRACT

In this paper we will study a non-autonomous piecewise linear difference equation which describes a discrete version of a single neuron model with a periodic internal decay rate. We will investigate the periodic behavior of solutions relative to the periodic internal decay rate. Furthermore, we will show that only periodic orbits of even periods can exist and show their stability character.

1. Introduction

In [1], the delayed differential equation

$$x'(t) = -g(x(t-\tau)) \tag{1}$$

is used as a model for a single neuron with no internal decay; where $g: \mathbf{R} \to \mathbf{R}$ is either a sigmoid or a piecewise linear signal function and $\tau \leq 0$ is a synaptic transmission delay. From (1) it is possible to obtain a discrete-time network of a single neuron model as follows:

$$x_{n+1} = \beta x_n - g(x_n), \;\; n = 0, 1, 2, \ldots, \tag{2}$$

where $\beta > 0$ is an internal decay rate, g is a signal function.

In [2], Eq. (2) was analyzed as a single neuron model where a signal function g is the following piecewise constant with McCulloch-Pitts nonlinearity:

$$g(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases} \tag{3}$$

Similar models of one, two or three neurons with constant internal decay rate β were studied in [3–12].

In this paper we will investigate the periodic character of solutions relative to the periodic internal decay rate; in particular, we will study the following non-autonomous piecewise linear difference equation:

$$x_{n+1} = \beta_n x_n - g(x_n), \tag{4}$$

* Corresponding author at: University of Latvia, Zeļļu 8, Rīga LV-1002, Latvia. Tel.: +37126470965.
E-mail addresses: inese.bula@lu.lv (I. Bula), michael.radin@rit.edu (M.A. Radin).

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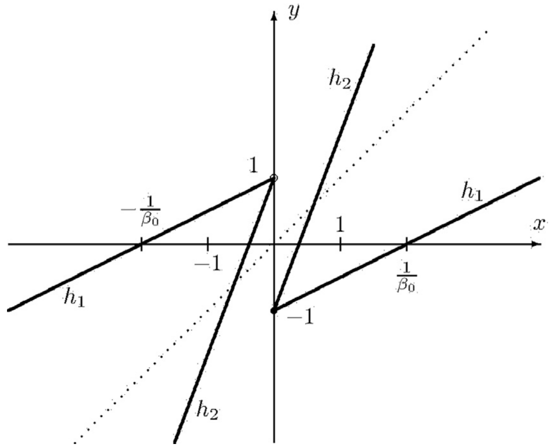

Fig. 1. Example of graphs of functions h_1, when $0 < \beta_0 < 1$, and $h_2(x)$, when $1 < \beta_1$.

where

$$\beta_n = \begin{cases} \beta_0, & \text{if } n \text{ is even,} \\ \beta_1, & \text{if } n \text{ is odd} \end{cases}$$

and g is in the form (3). So far the neuron model (2) has not been studied with a periodic internal decay rate β; therefore, we will have the opportunity to study this new idea in this paper. It is our goal to study the periodic cycles of (4) and their basin of attraction.

First we define

$$h_1(x) = \begin{cases} \beta_0 x - 1, & x \geq 0, \\ \beta_0 x + 1, & x < 0, \end{cases}$$

and

$$h_2(x) = \begin{cases} \beta_1 x - 1, & x \geq 0, \\ \beta_1 x + 1, & x < 0, \end{cases}$$

then it is possible that (4) be considered in the following form (see Fig. 1):

$$x_{n+1} = \begin{cases} h_1(x_n), & \text{if } n \text{ is even,} \\ h_2(x_n), & \text{if } n \text{ is odd.} \end{cases}$$

The points of the solution of (4) are located on the h_1 and h_2. We investigate nonlinear process.

The paper is organized as follows. In Section 2, we first give some basic concepts and definitions of difference equations used throughout the paper; then we analyze Eq. (4) and formulate the results about the periodicity and stability. At the end we will give some concluding remarks and future ideas.

2. Basic concepts and definitions of difference equations

To analyze the behavior of (4), it is essential to review some basic theory of difference equations (see [2,13,14]).

Consider a first order difference equation in the form

$$x_{n+1} = f(x_n), \quad n = 0, 1, \ldots, \tag{5}$$

where f: $\mathbf{R} \to \mathbf{R}$ is a given function. A solution of (5) is a sequence $(x_n)_{n \in \mathbf{N}}$ that satisfies (5) for all $n = 0, 1, \ldots$. If an initial condition $x_0 \in \mathbf{R}$ is given, then the *orbit* $O(x_0)$ of a point x_0 is defined as a set of points

$$O(x_0) = \{x_0,\ x_1 = f(x_0),\ x_2 = f(x_1) = f^2(x_0),\ x_3 = f(x_2) = f^3(x_0), \ldots\}.$$

Definition 1. A point x_s is said to be a fixed point of the map f, an equilibrium point or a stationary state of (5) if $f(x_s) = x_s$. Note that for a stationary state x_s the orbit consists only of the point x_s.

Definition 2. A stationary state of (5) is stable if

$$\forall \varepsilon > 0\ \exists \delta > 0\ \forall x_0 \in \mathbf{R}\ \forall n \in \mathbf{N}\ |x_0 - x_s| < \delta\ \Rightarrow\ |f^n(x_0) - x_s| < \varepsilon.$$

Otherwise, the stationary state x_s is *unstable*.