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A cutting plane projection method for bi-level area traffic control optimization with uncertain travel demand

Suh-Wen Chiou*

Department of Information Management, National Dong Hwa University, Da Hsueh Rd., Shou-Feng, Hualien, 97401, Taiwan

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ABSTRACT

A robust congestion pricing (ROPRICE) of signal-controlled road network is considered for equilibrium network flow with uncertain travel demand. A min–max bi-level program is presented to mitigate vulnerability of area traffic control road network against growing travel demand and reduce traffic congestion. A cutting plane projection approach (CPP) is proposed to effectively solve the ROPRICE problem with global convergence. Numerical computations are performed using various test road networks and comparisons are made with recently proposed heuristics. Computational results indicate that the proposed solution scheme can substantially achieve greater system performance as compared to other alternatives.

1. Introduction

For most urban road networks, severe traffic congestion would be incurred at signal-controlled junctions by road travelers as a result of inappropriate design of signal settings when uncertain travel demand is prevailing. Congestion pricing for system optimum flow has long been regarded as the most efficient instrument to mitigate traffic congestion and achieve efficient use of road network [1–7]. Among them, the marginal social cost pricing (MSCP) recognized as the first-best congestion pricing has been theoretically considered a most economically efficient tool to deal with congestion and make best usage of road network [8–14]. However, for some road networks, a second-best congestion pricing is pursued instead when only a subset of links can be tolled due to public acceptance and social and political restrictions. Ways of using mathematical programming to solve a constrained optimization problem of a second-best congestion pricing in a subset of road network links with certain travel demand have received ample attention [15–20]. Yang and Lam [15] proposed a sensitivity-analysis based algorithm (SAB) for simple congestion pricing problem in a general road network. Unfortunately, the relation between equilibrium flow and congestion pricing variables may not always be differentiable, which has been theoretically investigated in [21] and [22] and numerically illustrated in [23] and [24]. The combined problem of congestion pricing and signal settings for equilibrium flow can be formulated as a case of mathematical program with equilibrium constraints (MPEC) or MPCC (mathematical program with complementarity constraints) [25–27]. Lawphongpanich and Hearn [18] proposed a cutting constraint algorithm (CCA) approach to solving a second-best toll pricing problem. Using the concept of MPEC, constrained non-linear programming problems for second-best tolling optimization with fixed and variable travel demand were addressed. As it has been recognized widely from literature [28–30], the MPEC problem is generally a non-convex problem due to the implicit form in constraints. Chiou [12] introduced a projected subgradient approach to effectively solve a first-best congestion pricing problem for a general road network. More recently, Chiou [13,14] explored several heuristics respectively for variable travel demand in a general road network and for

* Tel.: +886 3 8633108; fax: +886 3 8633100.
E-mail address: chiou@mail.ndhu.edu.tw

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signal-controlled road networks with regard to certain travel demand. On the other hand, Sumalee [20] presented a GA-based heuristic to optimize a charging cordon in a general traffic network and gave promising results on finding optimal charging cordon design from empirical studies. The effectiveness of proposed GA-based heuristics has been demonstrated numerically at example test networks with considerable travel cost savings.

In order to mitigate vulnerability of system against uncertainty, there are basically two popular approaches to optimization under uncertainty. One is the approach of stochastic programming [31–33] modeling uncertain parameters as random variables with a priori probability distribution and optimizing the expected value of the objective function. For example, Gardner et al. [5] presented a stochastic mathematical program with equilibrium constraints for robust congestion pricing of transportation networks under uncertain demand and gave solution methods for robust prices which are resilient to demand variations. Li et al. [34] proposed a dynamic pricing for perishable products involving hybrid uncertainty in demand. Ban et al. [35] proposed a risk-neutral second-best toll pricing model to account for the possible non-uniqueness of user equilibrium solutions. Instead of knowing the probability distribution of uncertain parameters, the other approach, robust optimization, where one optimizes a worst possible case of a considered problem, has long been considered an effective tool to hedge against uncertainty [36–38]. In this regard, Lou et al. [39] presented a robust congestion pricing for a traffic road network under boundedly rational user equilibrium (BRUE). Chung et al. [2] also presented a dynamic congestion pricing with uncertain travel demand. Because of the non-linearity of the equilibrium constraint with respect to congestion pricing variables, most solution heuristics mentioned above can simply solve a non-convex and non-linear inequality constrained problem with congestion pricing only locally.

In this paper, a robust congestion pricing (ROPRICE) of a signal-controlled road network is presented following earlier work in Chiou [12]. In order to hedge against uncertain travel demand, a min–max model is proposed. A set of robust signals for a ROPRICE problem in the presence of uncertain travel demand can be determined against a worst-case that might be taken by the opponent in anticipation of road users' responding strategy via route choice. In this regard, the worst-case performance measure serves as an upper bound estimate. A cutting plane projection (CPP) is presented and numerical computations are performed using road networks of realistic size. The contributions made from this paper are summarized as follows. First, a ROPRICE problem is performed optimally to determine robust signal settings and link congestion pricing in the presence of uncertain travel demand. A Nash–Stackelberg equilibrium is established. The performance measure (PM), maximized with respect to a travel demand growth factor, is minimized with respect to signal-setting and link congestion pricing. A cutting plane projection is presented for a ROPRICE problem with uncertain demand. Computational results obtained indicate that proposed solution scheme can solve the ROPRICE problem successfully. As compared to deterministic solution for nominal travel demands using the MSCP, the proposed scheme achieved greater improvement while incurring a relatively slighter loss of optimality. The rest of the paper is organized as follows. Section 2 introduces a bi-level min–max model for a ROPRICE problem with uncertain travel demand. A Nash–Stackelberg solution can be effectively characterized by a tractable computation scheme proposed in Section 3. Numerical computations of proposed scheme are performed in Section 4. Conclusions for this paper and extensions of the proposed approach to topics of interest are briefly summarized in Section 5.

2. A ROPRICE problem with uncertain travel demand

In the presence of uncertain travel demand, a bi-level min–max model is introduced. Notation used for a ROPRICE problem with link congestion pricing is introduced first.

2.1. Notation

$G(N, L)$	a road network with node set N and link set L.
W	a set of origin–destination (OD) pairs.
R_w	a set of routes between OD pair w, $\forall w \in W$.
$\Psi = (\zeta, \theta, \phi)$	set of signal setting variables.
β	a set of link tolls.
λ_a	duration of effective green for link a, $\forall a \in L$.
$\lambda_{\min}$	minimum green.
τ_{jlm}	clearance time between the end of green for group j and the start of green for incompatible group l at junction m.
$\Omega_m(j, l)$	collection of numbers 0 and 1 for each pair of incompatible signal groups at junction m; where $\Omega_m(j, l) = 0$ if the start of green for signal group j proceeds that of l and $\Omega_m(j, l) = 1$, otherwise.
ρ_a	maximum degree of saturation for link a, $\forall a \in L$.
s_a	saturation flow on link a, $\forall a \in L$.
q	a matrix of travel demand for OD pairs.
μ	OD demand growth factor.
f	a vector of link flow, i.e. $f = [f_a]$, $\forall a \in L$.
h	a vector of route flow between OD trips, i.e. $h = [h_p]$, $\forall p \in R_w$, $\forall w \in W$.
δ	a link-route incidence matrix.
Λ	a OD-route incidence matrix.
$c(\Psi, \beta, f)$	a vector of link flow travel cost, i.e. $c(\Psi, \beta, \mu) = [c_a(\Psi, \beta, f)]$, $\forall a \in L$.
π	a vector of minimum travel cost between OD, i.e. $\pi = [\pi_w]$, $\forall w \in W$.