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Higher order iterative schemes for nonlinear equations using decomposition technique

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ABSTRACT

In this paper, we suggest and analyze some unique recurrence relations which can generate different classes of iterative methods for solving nonlinear equations using the system of coupled equations together with decomposition technique by using an auxiliary function. Various numerical examples are given to illustrate the efficiency and performance of the newly suggested methods. These new iterative methods may be viewed as an addition and generalization of the existing methods for solving nonlinear equations.

1. Introduction

It is well known that a wide class of problems, which arises in different disciplines of mathematical and engineering science can be studied by the nonlinear equation of the form $f(x) = 0$. Newton method and its modifications are being used to find the approximate solutions of the nonlinear equation, see [1–19] and the references therein. Abbasbandy [1] and Chun [3] have proposed and studied several one-step and two-step iterative methods with higher order convergence by using the Adomian decomposition technique [2]. In their suggested methods, the authors have used the higher order derivatives which are a severe drawback. To overcome this problem Noor and Noor [9,10] have considered another decomposition technique which does not involve the derivative of the Adomian polynomial. In this paper, an alternative decomposition technique is used to construct some new multi-step iterative methods for obtaining the approximate solution of nonlinear equations. Results obtained in this article suggest that this new technique of decomposition is a promising tool and can be considered as a substitute to the Adomian decomposition technique. The involvement of the arbitrary auxiliary function makes the technique much flexible for the derivation and implementation of the methods. This auxiliary function can provide the methods which overcome the drawbacks of the existing methods specially the Newton method and the methods which include the Newton method as predictor step. This is another motivation of the paper that the derived higher order methods also converge even if the derivative vanishes at any stage during the iterative process. Newly developed iterative methods include Newton method and Halley method and their variant forms as special cases. Several numerical examples are given to illustrate the efficiency and the performance of the new iterative methods. Our results can be considered as an important improvement and refinement of the previously known results.

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2. Iterative methods

In this section, some new iterative methods are suggested for solving nonlinear equations by using the system of coupled equations together with decomposition technique involving an auxiliary function. This auxiliary function diversifies the main recurrence relations for the best implementation of the methods to obtain the approximate solution of nonlinear equations. This technique purposes main iterative schemes to provide higher order convergent iterative methods.

Consider the nonlinear equation of the type

$$f(x) = 0. \tag{1}$$

Assume that α is a simple root of nonlinear Eq. (1) and γ is an initial guess sufficiently close to α. Let $g(x)$ be the auxiliary function, such that

$$f(x)g(x) = 0. \tag{2}$$

Eq. (2) can be re-written as a system of coupled equations and using Taylor series technique for the auxiliary function as:

$$f(\gamma)g(\gamma) + (x-\gamma)\left[\left\{\frac{f'(\gamma) + 4f'\left(\frac{\gamma+x}{2}\right) + f'(x)}{6}\right\}g(\gamma) + f(\gamma)g'(\gamma)\right] + h(x) = 0. \tag{3}$$

or

$$h(x) = f(x)g(\gamma) - f(\gamma)g(\gamma) - (x-\gamma)\left[\left\{\frac{f'(\gamma) + 4f'\left(\frac{\gamma+x}{2}\right) + f'(x)}{6}\right\}g(\gamma) + f(\gamma)g'(\gamma)\right]. \tag{4}$$

We can rewrite Eq. (4) in the following form:

$$x = \gamma - 6\left[\frac{f(\gamma)g(\gamma) + h(x)}{\left\{f'(\gamma) + 4f'\left(\frac{\gamma+x}{2}\right) + f'(x)\right\}g(\gamma) + 6f(\gamma)g'(\gamma)}\right]. \tag{5}$$

Let

$$x = c + N(x), \tag{6}$$

where

$$c = \gamma, \tag{7}$$

and

$$N(x) = -6\left[\frac{f(\gamma)g(\gamma) + h(x)}{\left\{f'(\gamma) + 4f'\left(\frac{\gamma+x}{2}\right) + f'(x)\right\}g(\gamma) + 6f(\gamma)g'(\gamma)}\right]. \tag{8}$$

Here $N(x)$ is a nonlinear function.

We now construct a sequence of higher order iterative methods by using the following decomposition technique, which is mainly due to Daftardar-Gejji and Jafari [5]. This decomposition of the nonlinear function $N(x)$ is quite different from that of Adomain decomposition.

The main idea of this technique is to look for a solution having the series form

$$x = \sum_{i=0}^{\infty} x_i. \tag{9}$$

The nonlinear operator N can be decomposed as

$$N(x) = N(x_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} x_j\right) - N\left(\sum_{j=0}^{i-1} x_j\right)\right\}. \tag{10}$$

Combining (6), (9) and (10), we have

$$\sum_{i=0}^{\infty} x_i = c + N(x_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} x_j\right) - N\left(\sum_{j=0}^{i-1} x_j\right)\right\}. \tag{11}$$