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On new subclass of meromorphic close-to-convex functions

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ABSTRACT

In the present paper, we introduce and investigate a certain new subclass $\mathcal{MK}(\lambda, t, A, B)$ of meromorphic close-to-convex functions. Some results such as inclusion relationship, coefficient inequality and some property for this class are derived. The results obtained here are extension of earlier known work.

1. Introduction

Let Σ denote the class of functions f of the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the punctured open unit disk $\mathbb{U}^* := \{z : z \in \mathbb{C}, 0 < |z| < 1\} =: \mathbb{U}\backslash\{0\}$, where $\mathbb{U}$ is an open unit disk. Let $\mathcal{P}$ denote the class of functions p of the form

$$p(z) = \frac{1}{z} + \sum_{n=1}^{\infty} p_n z^n$$

which are analytic in $\mathbb{U}$ and satisfy the condition $\mathrm{Re}(p(z)) > 0$ $(z \in \mathbb{U})$.

A function $f \in \Sigma$ is said to be in the class $\mathcal{MS}^*(\alpha)$ of meromorphic starlike functions of order α if it satisfies the inequality

$$Re\left(-\frac{zf'(z)}{f(z)}\right) > \alpha \ \ (z \in \mathbb{U}; 0 \le \alpha < 1).$$

Moreover, a function $f \in \Sigma$ is said to be in the class $\mathcal{MC}$ of meromorphic close-to-convex functions if it satisfies the condition

$$Re\left(-\frac{zf'(z)}{g(z)}\right) > 0 \ \ (z \in \mathbb{U}, g \in \mathcal{MS}^*(0) =: \mathcal{MS}^*).$$

Let $0 \le \alpha < 1$ and $\beta > 1$. A function $f \in \Sigma$ is said to be in the class $\mathcal{MS}^*(\alpha, \beta)$ of meromorphic starlike functions of order α if it satisfies the inequality

$$\alpha < Re\left(-\frac{zf'(z)}{f(z)}\right) < \beta \ \ (z \in \mathbb{U}).$$

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Let f and g be analytic in $\mathbb{U}$. Then f is subordinate to g, written $f \prec g$, if there exists an analytic function $\omega(z)$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ such that $f(z) = g(\omega(z))$. Indeed, it is known that

$$f \prec g \ (z \in \mathbb{U}) \Rightarrow f(0) = g(0) \text{ and } f(\mathbb{U}) \subset g(\mathbb{U}).$$

Furthermore, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence:

$$f \prec g \ (z \in \mathbb{U}) \Leftrightarrow f(0) = g(0) \text{ and } f(\mathbb{U}) \subset g(\mathbb{U}).$$

For some recent investigations on meromorphic functions and related functions, see (for example) the earlier works [1–4] and see the close-related recent works [5–7]. For some recent investigations on close-to-convex functions, see the earlier work [8,9].

Recently, Wang et al. [10] introduced and discussed the class $\mathcal{MK}$ of meromorphic functions $f \in \Sigma$ which satisfies the inequality

$$Re\left(\frac{f'(z)}{g(z)g(-z)}\right) > 0 \ (z \in \mathbb{U}). \tag{1.2}$$

where $g(z) \in \mathcal{MS}^*(\frac{1}{2})$.

More recently, by means of subordination, Sim and Kwon [11] discussed a subclass $\Sigma(A, B)$ of the class $\mathcal{MK}$. A function is said be in the class $\Sigma(A, B)$ if it satisfies the following subordination relation:

$$\frac{f'(z)}{g(z)g(-z)} \prec \frac{1 + Az}{1 + Bz} \tag{1.3}$$

where $-1 \le B < A \le 1$ and $g(z) \in \mathcal{MS}^*(\frac{1}{2})$. Here, the assumption that $g(z)$ is meromorphic starlike function of order $\frac{1}{2}$ makes the function $-zg(z)g(-z)$ meromorphic starlike. So, instead of $g(z)g(-z)$ in (1.2) and (1.3), we can consider $tg(z)g(tz)$, $0 < |t| \le 1$, because if $g(z) \in \mathcal{MS}^*(\frac{1}{2})$, then $tg(z)g(tz)$ is also a meromorphic starlike function, which motivates us to define a new subclass $\mathcal{MK}(\lambda, t, A, B)$ of meromorphic close-to-convex functions as follows.

Definition 1.1. A function $f \in \Sigma$ is said to be in the class $\mathcal{MK}(\lambda, t, A, B)$ if it satisfies the subordination condition

$$\frac{(1 + 2\lambda)f'(z) + \lambda z f''(z)}{tg(z)g(tz)} \prec \frac{1 + Az}{1 + Bz}$$

where $\lambda \ge 0$; $0 < |t| \le 1$, $-1 \le B < A \le 1$ and $g(z) \in \mathcal{MS}^*(\frac{1}{2}, \frac{\beta+1}{2})$ with $\beta > \frac{1}{\lambda} + 1(\lambda > 0)$.

In this paper, we aim at proving such results as inclusion relationships, coefficient inequalities, and property for the class $\mathcal{MK}(\lambda, t, A, B)$.

2. Properties of meromorphic starlike functions

We begin by proving the following result of meromorphic starlike functions.

Theorem 2.1. *Suppose that $\Phi \in \mathcal{MS}^*(\alpha_1, \beta_1)$ and $\Psi \in \mathcal{MS}^*(\alpha_2, \beta_2)$ with $0 \le \alpha_1 + \alpha_2 - 1 < 1$. Then*

$$zt_1t_2\Phi(t_1z)\Psi(t_2z) \in \mathcal{MS}^*(\alpha_1 + \alpha_2 - 1, \beta_1 + \beta_2 - 1)$$

where $0 < |t_1|, |t_2| \le 1$.

Proof. Let $\Phi \in \mathcal{MS}^*(\alpha_1, \beta_1)$ and $\Psi \in \mathcal{MS}^*(\alpha_2, \beta_2)$ with $0 \le \alpha_1 + \alpha_2 - 1 < 1$. Then

$$\alpha_1 < Re\left(-\frac{t_1z\Phi'(t_1z)}{\Phi(t_1z)}\right) < \beta_1$$

$$\alpha_2 < Re\left(-\frac{t_2z\Psi'(t_2z)}{\Psi(t_2z)}\right) < \beta_2$$

Next, we suppose that $h(z) = zt_1t_2\Phi(t_1z)\Psi(t_2z)$. Then, we easily find that

$$-\frac{zh'(z)}{h(z)} = -1 + \frac{-t_1z\Phi'(t_1z)}{\Phi(t_1z)} + \frac{-t_2z\Psi'(t_2z)}{\Psi(t_2z)}$$

It follows that $\alpha_1 + \alpha_2 - 1 < Re(-\frac{zh'(z)}{h(z)}) < \beta_1 + \beta_2 - 1$.

The proof of Theorem 2.1 is thus completed. □

Theorem 2.2. *Let $g(z) \in \mathcal{MS}^*(\frac{1}{2}, \frac{\beta+1}{2})$ and $0 < |t| \le 1$. Then $ztg(z)g(tz) \in \mathcal{MS}^*(0, \beta)$*

Proof. By setting $\Phi(z) = g(z)$, $\Psi(z) = g(tz)$, $\alpha_1 = \alpha_2 = \frac{1}{2}$, $t_1 = 1$ and $\beta_1 = \beta_2 = \frac{\beta+1}{2}$, we can get the assertion of Theorem 2.2. □

In order to derive our next results, we need the following lemmas.