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Hopf bifurcation and spatial patterns of a delayed biological economic system with diffusion

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ABSTRACT

In this paper, a delayed biological economic system which considers a plankton system with harvest effort on phytoplankton is proposed. By using the theory of partial functional differential equations, Hopf bifurcation of the proposed system with delay as the bifurcation parameter is investigated. It reveals that the discrete time delay has a destabilizing effect in the plankton dynamics, and a phenomenon of Hopf bifurcation occurs as the delay increases through a certain threshold. Then by numerical simulations the impact of delay, diffusion and economic interest on plankton system are explored. It is found that delay can cause system into chaos and can trigger the emergence of irregular spatial patterns via a Hopf bifurcation. Moreover, diffusion and economic profit can also affect the dynamic behavior of the system.

1. Introduction

Plankton acts as the base of the food chain in aquatic systems and plays a key role in the ecology of the sea and lakes. The study of plankton model has a great significance for the fisheries policy [1].

Since Scheffer [2] proposed the minimal model to describe the interaction of phytoplankton and zooplankton, many researchers have paid their attention to it [3–6]. The proposed model can be described as an ordinary differential equation (ODEs) as follows

$$\begin{cases} \dfrac{dP}{dt} = \dfrac{\alpha N}{H_N + N} P - \beta P^2 - \dfrac{VPZ}{H_P + P}, \\ \dfrac{dZ}{dt} = \dfrac{eVPZ}{H_P + P} - \delta Z - \dfrac{FZ^2}{H_Z^2 + Z^2}, \end{cases}$$

where P is phytoplankton biomass and Z is zooplankton. α is the maximum growth rate of population. N is the nutrient level of this system. H_N is the phytoplankton density at which specific growth rate becomes half of its saturation value. β is the competition coefficient of phytoplankton. V is the rate at which phytoplankton is grazed and it follows Holling type II functional response. e is the conversion coefficient from individuals of phytoplankton into individuals of zooplankton. F is the predation rate of zooplankton by fish population, which follows Holling type III functional response while δ is the mortality rate of predator. The interaction scheme on which the model is given in Fig. 1.

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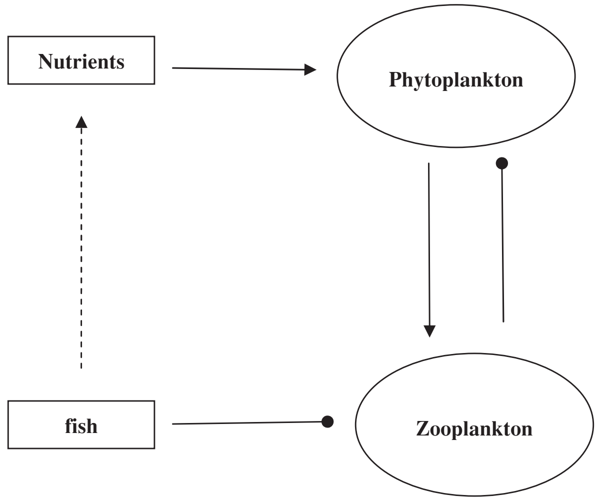

Fig. 1. The interactions that are incorporated in the model. Circles stand for negative effects and arrows denote positive effects.

The spatio-temporal dynamics of interacting biological, economic or social components can generate numerous local and spatially distributed effects far from equilibrium point, such as excitability, steady-state multiplicity, regular and irregular oscillations, and pulses as well as stationary spatial patterns [7–10]. In real life, the phytoplankton and zooplankton are always moving, that is to say, the population densities have become space and time dependent [11]. For this reason, the spatio-temporal dynamics of plankton model is necessary to be studied. So far, there have been a lot of excellent papers about plankton dynamics with diffusion in a plankton system [12–15]. Most of them mainly focus on determining the diffusion-driven spatiotemporal patterns. It is well known that irregular spatiotemporal patterns have important implications for population dynamics [16], and are thought to be ecologically relevant because they can emerge under less restrictive conditions compared to the Turing pattern. The basic properties conform well to the spatial irregularity of species distribution which is typically observed in nature. Spatiotemporal chaos is thought to have important implications for ecosystem functioning not only in terms of their predictability but also by affecting straightforwardly species persistence and ecosystem's stability with respect to unfavorable environmental changes [17]. In [14] the authors incorporated diffusion terms into the minimal model and considered the following reaction-diffusion model

$$\begin{cases} \dfrac{\partial P}{\partial t} = d_1 \Delta P + \dfrac{\alpha N}{H_N + N} P - \beta P^2 - \dfrac{VPZ}{H_P + P}, \\ \dfrac{\partial Z}{\partial t} = d_2 \Delta Z + \dfrac{eVPZ}{H_P + P} - \delta Z - \dfrac{FZ^2}{H_Z^2 + Z^2}. \end{cases}$$

The authors studied the reaction diffusion pattern formation mechanism of the model and found that the reaction diffusion model exhibits spatiotemporal chaos causing plankton patchiness in marine system.

On the other hand, the interaction between phytoplankton and zooplankton will not be essentially instantaneous in a real ecological context. Mukhopadhyay and Bhattacharyya [18] extended the "minimal" model to include the gestation delays

$$\begin{cases} \dfrac{dP}{dt} = rP\left(1 - \dfrac{P}{k}\right) - \dfrac{gPZ}{h_1 + P}, \\ \dfrac{dZ}{dt} = \dfrac{egZP(t-\tau)Z}{h_1 + P(t-\tau)} - mZ - \dfrac{FZ^2}{h_2^2 + Z^2}. \end{cases}$$

They analyzed the delayed model for its stability and bifurcation aspects. Hillary and Bees [3] compared the dynamics of the plankton communities with and without the delay associated with a zooplankton maturation period and found that time delay can lead to progressively replacing regular plankton dynamics by chaos. There are also other causes of delays in population dynamics at various time scales [5,7,11–14,18]. Consequently, delay needs to be considered.

Motivated by the above discussions, it is important to study the model with delay and diffusion in a marine ecosystem. Furthermore, we have noted that the ecological system is often deeply perturbed by human exploiting activities over recent years [5,19,20]. For the view of human needs, the exploitation of ecological resources and harvest of population are commonly carried out in the fields of wildlife, fishery and forestry management [14]. As a consequence, people are facing the problems about shortage of resource and worsening environment. For human being's sustainable development, many researchers have developed great interests in studying biological systems with harvest terms. There have been many exciting achievements for this. Kar [21] considered a prey–predator model with harvesting. Chakraborty et al. [22] formulated a control problem and described a corresponding optimality system that characterizes the (continuous) optimal control solution. Liu et al. [23] investigated Hopf