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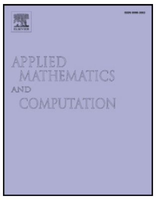

Permanence and asymptotical behavior of stochastic prey–predator system with Markovian switching[☆]

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ABSTRACT

In this paper, we investigate the stochastic permanence and extinction of a stochastic ratio-dependent prey–predator model controlled by a Markov chain. In the permanent case we estimate the superior limit and the inferior limit of the average in time of the sample path of the solution. The boundaries are related to the stationary probability distribution of the Markov chain and the parameters of the subsystems. Finally, we illustrate our main results by two examples and some numerical experiments.

1. Introduction

The dynamic relationship between predators and their preys has been universal existence in both ecology and mathematical ecology [13]. Lotka and Volterra [30–32,48] investigated the famous prey–predator model, respectively,

$$\begin{cases} \dfrac{dx}{dt} = a_1x - c_1xy, \\ \dfrac{dy}{dt} = -a_2y + c_2xy, \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ represent the population density of prey species (for example, rabbits) and predator species (for example, foxes) at time t, parameters a_1, a_2, c_1, c_2 are all positive constants describing the interaction of the two species. Intra-species conflict and competition are part of nature in most species, then prey–predator Lotka–Volterra model with intra-species competition becomes

$$\begin{cases} \dfrac{dx}{dt} = x(a_1 - b_1x - c_1y), \\ \dfrac{dy}{dt} = y(-a_2 - b_2y + c_2x), \end{cases} \tag{1.2}$$

where the positive constants b_1, b_2 represent the effect of one species on the other. It is well known that the solution of (1.2) is asymptotically stable.

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Both models (1.1) and (1.2) assume the prey biomass is enough and an individual predator consumes the prey with functional response of type c_1x . When predators are faced with increasing local density of their prey, they often respond by changing their consumption rate. The idea of a functional response was introduced by Solomon and explored in detail by Holling later. Holling [16–18] recognized three types of functional response, i.e., Hollings type I of form αx, Hollings type II of form $\frac{\alpha x}{\beta+x}$ and Hollings type III of form $\frac{\alpha x^2}{\beta+x^2}$. Furthermore, Bazykin et al. [8] (page 67) and Mcgehee et al. [41] investigated the rich dynamics of prey–predator model with intraspecific competition basing on Holling type II response function, gave the interesting coexistence phenomenon of two attractors and analyzed the bifurcation of limit circles and equilibriums.

When predators have to search for food and therefore have to share or compete for food, a ratio-dependent functional response is more reasonable and has been strongly supported by numerous fields, laboratory experiments and observations [1–4,14]. Based on the Holling type II function, Arditi and Ginzburg [1] first proposed a ratio-dependent functional response of form $\frac{\alpha x}{x+ey}$ and obtained the following ratio-dependent prey–predator model

$$\begin{cases} \dfrac{dx}{dt} = a_1x - b_1x^2 - \dfrac{c_1xy}{x+ey}, \\ \dfrac{dy}{dt} = -a_2y + \dfrac{c_2xy}{x+ey}, \end{cases} \tag{1.3}$$

where parameters c_1, c_2, e are all positive constants, representing capturing rate, conversion rate and half capturing saturation constant, respectively. A major mathematical difficulty with ratio-dependent functional response is that the concerned model fails to satisfy the continuity condition at origin [34]. Recently, many authors have drawn their attention on (1.3) and discussed its rich dynamical behavior, for example, [9,10,19,26].

The papers mentioned above are all deterministic models which do not incorporate the effect of either environmental fluctuations or demographic stochasticity. In fact, both environmental fluctuations and demographic stochasticity are important components for population systems. May [40] has pointed out that all of parameters in the population model are not constants but exhibit random fluctuations. Thus most of natural phenomena do not follow strictly deterministic laws but rather oscillate randomly around some average values [22]. Therefore stochastic models play significant roles to study the dynamics of interacting populations under realistic situation. However, there is no unified approach to formulate a stochastic model. Mandal and Banerjee [34] divided the available formalisms into two broad classes, one is white noise added models and the other is Markovian chain driven models (discrete or continuous time).

Recently, many authors have paid their attention to stochastic prey–predator models with white noise and revealed how the noise affect the population systems, for example, [6,20,21,23,34,44]. Especially, Arnold et al. [6] pioneered the investigation of stochastic prey–predator model

$$\begin{cases} \dfrac{dx}{dt} = x(a_1 - b_1x - c_1y)dt + \sigma_1xdB_1(t), \\ \dfrac{dy}{dt} = y(-a_2 - b_2y + c_2x)dt + \sigma_2ydB_2(t), \end{cases} \tag{1.4}$$

and studied the properties of sample paths. Basing on Eq. (1.3), Ji et al. in [20,21,23] investigated stochastic differential equation (SDE)

$$\begin{cases} dx = x\left[a_1 - b_1x - \dfrac{c_1y}{x+ey}\right]dt + \sigma_1xdB_1(t), \\ dy = y\left[-a_2 + \dfrac{c_2x}{x+ey}\right]dt + \sigma_2ydB_2(t). \end{cases} \tag{1.5}$$

They obtained the stationary distribution and some asymptotic properties. Moreover, Nguyen and Ta in [44] introduced the intra-specific competition into (1.5),

$$\begin{cases} dx = x\left[a_1 - b_1x - \dfrac{c_1y}{x+ey}\right]dt + \sigma_1xdB_1(t), \\ dy = y\left[-a_2 - b_2y + \dfrac{c_2x}{x+ey}\right]dt + \sigma_2ydB_2(t), \end{cases} \tag{1.6}$$

then they considered the corresponding non-autonomous stochastic system and estimated the upper-growth rates and the exponential death rates of the population. As a matter of fact, many authors have studied the prey–predator system perturbed by the white noise but we don't mention them in details.

Let us now take a further step to add the other type of environmental noise. This noise means a random switching between two or more environmental regimes distinguished by factors such as nutrition or rainfall [11,42]. The stochastic differential equations driven by a continuous-time Markov chain have been used to model the population systems [27,28,33,38,47,49] with this type of noise. Suppose the Markov chain $r(t)$ on the state space $S = \{1, 2, \dots, N\}$ controls the switching between the environmental regimes. Then the prey–predator model with two types of noise can therefore be described by the following stochastic