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Adaptive dynamics analysis of a predator–prey model with selective disturbance☆

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ABSTRACT

Evolution problem is always a hot topic in the mathematical biology field. In this paper, we investigate the evolutionary effects of selective disturbance on an evolving trait (e.g. body size and maturation age) of the predator individuals in one-predator two-prey community. By using methods of adaptive dynamics and population dynamics we construct an invasion fitness function and obtain the conditions for evolutionary branching and evolutionary stability under selective disturbance in both monomorphic and dimorphic populations. We further conduct a size-selective disturbance function founded on chi-square distribution to study evolutionary stable coexistence, and considering the evolutionary branching and evolutionary stability by using theoretic analysis and numerical simulations. The evolutionary results from a biological point of view show that (1) two strategies could gradually evolve to form a single ancestral strategy, moreover, higher levels of polymorphism cannot build up during evolution, that is, following first evolutionary branching two species will eventually evolve into two generalist species and reach an evolutionary stable coexistence; (2) smaller disturbance could touch off higher levels of dimorphism during evolution, while large disturbance can go against evolutionary branching and advance evolutionary stability.

1. Introduction

Ecosystems have always been disturbed by artificial selection, such as breeding and harvesting. Disturbances from human could change the wild species and thus induce changes in the biological processes as well as the ecological stability [1]. It has been confirmed by many experts that artificial selection harvesting for mature individuals with large value of the trait drives evolution towards shorting maturation period or decreasing body size in many species, like fishes, animals and plants [2–4]. Existed research works about harvest may lead to changes in trait value mainly focus on experimental analysis or numerical simulations, but work on the evolutionary adaptive dynamics is still very little.

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Predator–prey models have widely attracted the focus of the people since Lotka–Volterra model was accepted by biologist. The population dynamics of the predator–prey models has been extensively investigated [5–12]. For evolution problem of predator–prey model, most of the work is to investigate how and why foraging-related traits evolve in a community comprising the populations of one evolving predator and prey [13–19]. The objective of this paper is to explore the impacts of selective disturbance in predators on evolutionary changes in adaptive dynamics of a one-predator two-prey community. Here, we regard the value of individuals (e.g. body size and maturation age) as the phenotype trait and consider the effect on predators with selective disturbance during evolution, then explore the continuously stable strategy and evolutionary branching. A series of work assumed that harvesting can promote the evolution towards smaller body size, however, for breeding, research works are still rare, therefore we are looking forward to an opposite effect.

The rest of the paper is organized as follows. In the next section, we present the one-predator two-prey model with selective disturbance and derive the invasion fitness for mutant predators in monomorphic environments. Moreover, conditions for evolutionary branching and evolutionary stable strategy are also discussed. In Section 3, we discuss the dimorphic coexistence and coevolution of population model with two resident predators. The influence of selective disturbance on adaptive dynamics and numerical simulations are also presented in Sections 2 and 3, respectively. Then conclusions and a brief discussion are given in Section 4.

2. Monomorphic evolutionary dynamics

In this section, we start with a population dynamical model for an evolving predator with trait x feeding on two prey species, and consider the conditions for evolutionary branching. For this purpose, we are going to obtain an invasion fitness function to investigate the evolutionary dynamics.

2.1. The model

We investigate the evolution of a single continuous trait on predator such as body size or maturation period. According to the common sense that individuals with large trait value are more easily to be harvested, we assume that large value of the trait means high harvesting rate. Moreover, the two prey species are homogeneously distributed, they may compete with each other without affecting the predator's functional response to the other prey.

Let $N_i (i=1,2)$ denote the population densities of prey i and P denotes the population density of predators at time t with trait x. Therefore, the one-predator two-prey dynamics is given by

$$\begin{cases} \dfrac{dN_1}{dt} = r_1 N_1\left(1 - \dfrac{N_1 + a_1 N_2}{K_1}\right) - \alpha_1(x) N_1 P, \\ \dfrac{dN_2}{dt} = r_2 N_2\left(1 - \dfrac{N_2 + a_2 N_1}{K_2}\right) - \alpha_2(x) N_2 P, \\ \dfrac{dP}{dt} = \varepsilon\alpha_1(x) N_1 P + \varepsilon\alpha_2(x) N_2 P - dP + H(x)P, \end{cases} \tag{1}$$

where r_i $(i=1,2)$ represent the intrinsic growth rates of prey population i, K_i $(i=1,2)$ is the biggest environmental intake capacity of prey i, a_i $(i=1,2)$ denote interspecific competitive effects between two prey species, α_i $(i=1,2)$ represent the per capita capture rates on prey i, ε is the conversion efficiency of ingested prey into new predators, d is the per capita death rate of predator population.

According to the fact that predator's capture rate depends on it's own phenotype trait x and the phenotype trait of prey species, since those two phenotype traits belong to different species, then they are irrelevant. The objective of this paper is to explore the evolution change on phenotype trait of predators, then we assume that the average trait value on prey population is constant. Predator's capture rate functions are indicated as follows [20–24]:

$$\alpha_1(x) = p(x)e^{-(x-a)^2/\delta_\alpha^2}, \alpha_2(x) = 1 - p(x)e^{-(x-a)^2/\delta_\alpha^2}, \tag{2}$$

where $p(x) = \frac{k\sin(14x)+14x}{60}$, $\alpha_1(0)=0, \alpha_2(0)=1, \alpha_1'(x) > 0, \alpha_2'(x) < 0$, a and δ_α are positive constants and $x \in [0,1]$.

$H(x)$ is the disturbance rate of the predator population P. $H(x) = h(x) \geq 0$ stands for breeding of the predator population, while $H(x) = -h(x) < 0$ for harvesting. Without the loss of generality, we assume the chi-square distribution $h(x)$ satisfies the following two properties (i) $h(x) \geq 0$, (ii) $h(x)$ is a continuous and increasing function with respect to x.

Let $h(x) = 1 - (1+\delta x)e^{-\delta x}$ (see [25]), where δ is the harvesting or breeding power of the predator population. The concave–convex function $h(x)$ respect to x is shown in Fig. 1.

Let the right hand sides of system (1)–0, when

$$\begin{cases} K_1 r_2 \alpha_1^2(x) + K_2 r_1 \alpha_2^2(x) > (a_1 r_1 K_2 + a_2 r_2 K_1)\alpha_1(x)\alpha_2(x), \\ K_2 r_1 \alpha_2(x)[\varepsilon K_1 \alpha_2(x) + a_1 H(x)] + r_2 d K_1 \alpha_1(x) > K_1 r_2 \alpha_1(x)[\varepsilon K_2 \alpha_2(x) + H(x)] + r_1 a_1 d K_2 \alpha_2(x), \\ K_1 r_2 \alpha_1(x)[\varepsilon K_2 \alpha_1(x) + a_2 H(x)] + r_1 d K_2 \alpha_2(x) > K_2 r_1 \alpha_2(x)[\varepsilon K_1 \alpha_1(x) + H(x)] + r_2 a_2 d K_1 \alpha_1(x), \\ \varepsilon[K_1\alpha_1(x) + K_2\alpha_2(x)] + a_1 a_2 d + H(x) > \varepsilon[a_1 K_2 \alpha_1(x) + a_2 K_1 \alpha_2(x)] + a_1 a_2 H(x) + d, \end{cases} \tag{3}$$