Applied Mathematics and Computation 266 (2015) 1050–1069

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Improved stability analysis of uncertain neutral type neural networks with leakage delays and impulsive effects

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ARTICLE INFO

Keywords:
Neural network
Neutral type
Lyapunov functional
Leakage delay
Impulse

ABSTRACT

This paper focuses on the stability analysis for neural networks of neutral type with leakage delays and impulsive effects. The discrete delays are assumed to be time-varying and belong to a given interval, which means that the lower and upper bounds of interval time-varying delays are available. By utilizing the Lyapunov functional method, Jensen's integral inequality and introducing some free-weighting matrices, some new delay-derivative-dependent stability criteria are established for the neutral type neural network. The obtained stability criteria are stated in terms of linear matrix inequalities. Finally, numerical examples are given to illustrate the effectiveness and reduced conservatism of the proposed results over the existing ones.

1. Introduction

The research of neural networks has received considerable attention during the past few decades. In particular, neural networks have been extensively implemented in various applications, such as automatical control, signal processing, solving certain optimization problems, machine learning, and so on. Moreover, it is well known that time delays are unavoidable in hardware implementations, due to the finite switching speed of amplifiers. The existence of time delays can destroy the stability and/or weaken the performance of the essential neural networks. So the issue of stability analysis of neural networks with time delays attracts many researchers and a large number of stability criteria have been reported in the open literature [29,34,38,48]. It should be noted that all these stability criteria can be classified into two categories, that is, delay-independent ones [1] and delay-dependent ones [2–4,32]. Delay independent stability conditions do not take the delay size into consideration, and they are often conservative for systems with small delays. Therefore, the investigation on the stability of neural networks with delay-dependent conditions has been the highlight in this field.

Also, many dynamical neural networks are described with neutral functional differential equations that include neutral delay differential equations as their special case. These neural networks are called neutral neural networks or neural networks of neural-type. Recently, some results for neural networks of neural-type have been derived in the literature [5,6,30,31,46].

As pointed out by Gopalsamy in [7], the time delay in the stabilizing negative feedback term has a tendency to destabilize a system. Like the traditional time delays, the leakage delays also have a great impact on the dynamics of neural networks and

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http://dx.doi.org/10.1016/j.amc.2015.06.030
0096-3003/

many works have appeared in the literature, see [8–12]. In [8], Gopalsamy initially investigated the BAM neural networks with leakage delays and obtained some sufficient conditions to guarantee the existence and global stability of a unique equilibrium point by employing M-matrices theory. Then based on this work, Peng [9] further discussed the existence and global stability of periodic solutions for BAM neural networks with leakage delays by using the continuation theorem in coincidence degree theory and the Lyapunov functional. In [10,11], the authors studied the equilibrium point of two classes fuzzy neural networks with delays in leakage terms. By using the topological degree theory, delay-dependent stability conditions of neural networks of neutral type with time delays in the leakage term were proposed in [12]. Therefore, it is valuable to investigate the stability analysis of neural networks with time delays in the leakage term. Recently, there have appeared a few works on the stability analysis of neural networks with time delays in the leakage term in the literature [13–17].

In addition, many physical systems undergo unexpected changes at certain moments due to instantaneous perturbations, which leads to impulsive effects. It is worth pointing out that neural networks are often subject to impulsive perturbations that in turn affect dynamical behaviors of the system. It frequently occurs in fields such as economics, mechanics, electronics, telecommunications, medicine, biology, etc. Therefore, it is necessary to consider impulsive effects to the problem of neural networks to reflect more realistic dynamics and many results have been reported for continuous-time and discrete-time neural networks [18–25]. Unfortunately, due to some theoretical and technical difficulties, up to now, the stability problem of neural networks of neutral type with the effects of leakage delays via impulsive control has not been addressed, which is still an open problem and remains challenging. This situation encourages our present research.

Motivated by the above discussion, in this paper we study the stability issue of impulsive neutral type neural networks with leakage delays. By using the Lyapunov functional, and free weighting matrix method, some sufficient conditions that depend on the delays for stability criteria are obtained in terms of LMIs, which can be readily verified by using the standard numerical software. Moreover, numerical examples are provided to illustrate the effectiveness of the proposed criteria.

Notations. Throughout this paper, $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The notation $X \geq 0$ (respectively, $X > 0$), where X is symmetric matrices, means that X is positive semi-definite (respectively, positive definite). The subscript T denotes the transpose of the matrix.

2. Problem formulation

Consider the following neutral type neural networks with both leakage delays and impulsive perturbations

$$\begin{aligned} dx(t) &= -Ax(t-\delta) + W_1 f(x(t)) + W_2 f(x(t-\tau(t)) + W_3\dot{x}(t-h(t)), \quad t \neq t_k, \\ \Delta x(t_k) &= -N_k\left\{x(t_k^-) - A\int_{t_k-\delta}^{t_k} x(s)ds\right\}, \quad t = t_k, \quad k \in \mathcal{Z}_+, \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathcal{R}^n$ is the state vector associated with the neurons, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T$ is the activation function. The matrix $A = \text{diag}(a_1, a_2, \ldots, a_n)$ is a diagonal matrix with positive entries $a_i > 0$. W_1, W_2, W_3 are the interconnection matrices representing the weight coefficients of the neurons. $N_k \in \mathcal{R}^{n\times n}$, $k \in \mathcal{Z}_+$ denotes the impulsive matrix. $\delta \geq 0$ denotes the constant leakage delay. The discrete delay $\tau(t)$ and the neutral delay $h(t)$ are assumed to satisfy

$$\begin{aligned} &0 \leq \tau_1 \leq \tau(t) \leq \tau_2, \quad \dot{\tau}(t) \leq \tau_d < 1; \quad 0 < h(t) \leq h, \quad \dot{h}(t) \leq h_d < 1, \\ &\tau_{12} = \tau_2 - \tau_1, \quad \tau_s = \frac{1}{2}(\tau_2^2 - \tau_1^2), \quad \tau_c = \frac{\tau_2^3 - \tau_1^3}{6}, \end{aligned} \tag{2}$$

where $\tau_1, \tau_2, \tau_d, h_d$ and h are constants. The initial condition associated with model (1) is given by

$$x(t) = \phi(t), \quad \forall t \in [-\max\{\delta, \tau_2, h\}, 0].$$

Throughout this paper, we assume that the following conditions are satisfied:

(H1) For any $j \in 1, 2, \ldots, n$, $f_j(0) = 0$ and there exist constants F_j^- and F_j^+ such that

$$F_j^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_j^+, \tag{3}$$

for all $\alpha_1, \alpha_2 \in \mathcal{R}$ and $\alpha_1 \neq \alpha_2$.

(H2) The impulsive time instant t_k satisfy $0 = t_0 < t_1 < \cdots < t_k \to \infty$ and $\inf_{k\in Z_+}\{t_k - t_{k-1}\} > 0$.

Next, we present some preliminary lemmas, which are needed in the proof of our main results.

Lemma 2.1. *For any symmetric positive-definite constant matrix* $R = \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0$, *where* $R_1, R_2, R_3 \in \mathcal{R}^{n\times n}$, *and* $0 < h(t) \leq h$, *if there exists a vector function* $\dot{x}(\cdot) : [0, h] \to \mathcal{R}^n$ *such that the following integration is well defined, then we have*

$$-h(t)\int_{t-h(t)}^{t} \eta^T(s)R\eta(s)ds \leq \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \int_{t-h(t)}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 & -R_2^T \\ * & -R_3 & R_2^T \\ * & * & -R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \int_{t-h(t)}^{t} x(s)ds \end{bmatrix}$$

where $\eta(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix}^T$.