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A probabilistic estimation and prediction technique for dynamic continuous social science models: The evolution of the attitude of the Basque Country population towards ETA as a case study

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A R T I C L E I N F O

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A B S T R A C T

In this paper, a computational technique to deal with uncertainty in dynamic continuous models in Social Sciences is presented. Considering data from surveys, the method consists of determining the probability distribution of the survey output and this allows to sample data and fit the model to the sampled data using a goodness-of-fit criterion based on the χ^2-test. Taking the fitted parameters that were not rejected by the χ^2-test, substituting them into the model and computing their outputs, 95% confidence intervals in each time instant capturing the uncertainty of the survey data (probabilistic estimation) is built. Using the same set of obtained model parameters, a prediction over the next few years with 95% confidence intervals (probabilistic prediction) is also provided. This technique is applied to a dynamic social model describing the evolution of the attitude of the Basque Country population towards the revolutionary organisation ETA.

1. Introduction

Uncertainty quantification in dynamic continuous models is an emerging area [1]. Because of the numerous complex factors that usually involve social behavior, it is particularly appropriate the consideration of randomness in this kind of models. In practice, the introduction of randomness in continuous models can be done using different approaches. Stochastic differential equations of Itô-type consider uncertainty through a stochastic process called white noise, i.e., the derivative of a Wiener process. As a consequence, this approach limitates the introduction of uncertainty to a Gaussian process whose sample trajectories are somewhat irregular since they are nowhere differentiable. A more convenient approach in social modeling is to permit that input parameters can become random variables and/or stochastic processes and, therefore can follow other type of probability distributions apart from Gaussian. This approach leads to continuous models usually referred to as random differential equations (r.d.e.'s). In dealing with r.d.e.'s, generalised Polynomial Chaos (gPC) is one of the most fruitful methods [2,3].

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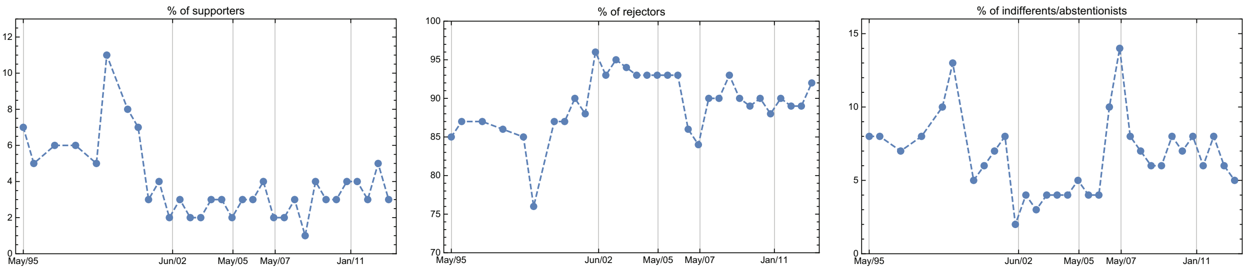

Fig. 1. Percentage of Basque Country population with an attitude of support, rejection or abstention towards ETA since May 1995 until November 2012. Vertical lines correspond to remarkable dates: in Jun 2002 the Law of Political Parties was passed and left-wing nationalist political parties were outlawed because their proven relation with ETA; in May 2005 the Spanish Parliament approved the possibility the Government to support dialogue with ETA; in May 2007 left-wing nationalist parties could present candidates again; in Jan 2011 ETA announced a permanent cease-fire. Observe that large jumps in the rejection population correspond to large jumps in the Abstention population, but in the opposite sense. Supporting population remains with minor variations since the Law of Political Parties passed.

Most of the existing methods and techniques, assume that input model parameters have known standard probability distributions. In general, setting the probability distribution of the model parameters, standard or empirical, is a crucial and difficult task currently under study which is required for uncertainty modeling approaches.

Also, the computation is an important issue in dealing with uncertainty. For instance, gPC technique may not be affordable when the number of model parameters which are assumed random variables, increases, or the interval where the mean and the standard deviation are valid may be very short [4]. It may turn these techniques inappropriate for modeling real problems.

On the other hand, if we consider that there is not information available for setting the model parameters probabilistic distribution, techniques as bootstrapping [5,8] or Bayesian [6] are other useful approaches. Related to these statistical techniques, in this paper we propose a computational approach where the data, retrieved from surveys, play a fundamental role to introduce the uncertainty, in estimation and prediction, from the very beginning. This probabilistic approach is applied to a model describing the evolution of the attitude of the Basque population towards the revolutionary organisation ETA [7] presented in [8]. In this latter paper, the authors apply the bootstrapping technique to the model in order to deal with uncertainty with the system of differential equations, because we do not assume error in the data. As we will see in the present paper, the uncertainty comes with the data (data sample error) and we will propose new techniques to make the model captures the data uncertainty.

The paper is organised as follows. In Section 2, we summarise the model building described in [8]. In Section 3 we propose a technique which will allow us to obtain a set of model parameters that provide 95% confidence intervals (95% CI) for each time instant such that the data uncertainty is captured. We will call this technique *probabilistic estimation*. With the set of parameters obtained in Section 3, in Section 4 we obtain a *probabilistic prediction* of the attitude towards ETA of the people of the Basque Country over the next four years. In Section 5, we discuss the results and present the conclusion.

2. Model building

In [8], a deterministic mathematical model was introduced, and series data were retrieved from the Euskobarometro of November 2012 on the attitude of the Basque Country population towards ETA [9, Table 20]. The eight types of attitudes towards ETA that appear in the Euskobarometro (Total support; Justification with criticism; Goals yes/Means no.; Before yes/Not now; Indifferent; ETA scares; Total rejection; No answer) were simplified to only three (support; rejection; abstention) and allowed us to divide Basque Country population into the following three subpopulations, time t in years (see [8] for more details):

- **Supporters.** $A_1(t)$ denotes the percentage of people in the Basque Country who have an attitude of support towards ETA at the time instant t,
- **Rejectors.** $A_2(t)$ denotes the percentage of people in the Basque Country who have an attitude of rejection towards ETA at the time instant t,
- **Abstentionists.** $A_3(t)$ denotes the percentage of people in the Basque Country whose attitude towards ETA is not defined (indifferent), abstain or simply they do not want to declare their opinion, at the time instant t.

Data in these three groups appear, in percentages, in Fig. 1 from May 1995 until November 2012. In May 2005 the Spanish Parliament approved the possibility the Government to support dialogue with ETA. This fact has been considered as a substantial change in the anti-terrorist policy. This policy is still in force and it justifies that we have chosen this time instant as our model initial condition. In Table 1, the figures in percentages of each subpopulation from May 2005 to November 2012 are presented.

As it was shown in [8], the following system of nonlinear differential equations describes the evolution of attitudes towards ETA in the Basque Country over the time:

$$
\begin{aligned}
A_1'(t) &= \beta_{21}A_2(t)A_1(t) - \beta_{12}A_1(t)A_2(t) + \beta_{31}A_3(t)A_1(t) - \beta_{13}A_1(t)A_3(t),\\
A_2'(t) &= \beta_{12}A_1(t)A_2(t) - \beta_{21}A_2(t)A_1(t) + \beta_{32}A_3(t)A_2(t) - \beta_{23}A_2(t)A_3(t),\\
A_3'(t) &= \beta_{13}A_1(t)A_3(t) - \beta_{31}A_3(t)A_1(t) + \beta_{23}A_2(t)A_3(t) - \beta_{32}A_3(t)A_2(t).
\end{aligned}
$$