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The existence and global exponential stability of periodic solution for a neutral coupled system on networks with delays

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ABSTRACT

In this paper, we establish sufficient conditions for the existence and global exponential stability of periodic solution to a type of neutral coupled system on networks with delays. The key to prove the existence of periodic solutions is using the combined method of graph theory, coincidence degree theory and Lyapunov functional method. And the sufficient conditions are easy to be checked. Finally, a numerical simulation is carried out to show the correctness of our main results.

1. Introduction

In the real world, periodic motion is common in many processes such as in biology, the wave vibration, climate changes in four seasons, physics and so on. And many of these phenomena can be modeled by differential equations. For example, certain population ecology, distributed networks, etc., can be modeled by neutral differential equations with delays. So it is important to study the dynamics of neutral differential equations with delays. The existence and stability of periodic solution of neutral differential equations are interesting and important topics. Many results on the existence of periodic solutions of neutral systems have been reported, see [1–6]. In papers [5,6], by using coincidence degree theory and Lyapunov functional method, the authors studied the existence and global attractivity of periodic solution for neutral functional differential systems with delays.

On the other hand, in recent years, coupled systems on networks (CSNs) have also been studied extensively due to their applications in many ways such as in artificial complex dynamical systems, neural networks, the spread of infectious diseases and so on (see Refs. [7–13] and references therein). Studying the dynamical behaviors of CSNs, especially the stability, is one of the dominant themes, and many research papers have been reported (see Refs. [14–25] and references therein). For example, papers [14–19] were about stability analysis for coupled systems of differential equations on networks, Refs. [20–23] investigated the stability of coupled systems with time delays on networks, and Refs. [24,25] discussed the stability of stochastic coupled systems on networks. Up to now, most of the work is about the global stability of equilibrium point, and the existence and global stability of periodic solution for CSNs with periodic coefficients are investigated far less. Studying such problem has been difficult due to the complexity of coupled systems on networks.

Stimulated by above facts, in this paper, we study the periodic solutions for a type of neutral coupled system on networks with delays (NCSNDs). In reference to the existing results (for example, [5,6]), the contributions and novelties are as follows:

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1. Sufficient conditions for existence, uniqueness and global exponential stability are obtained by applying graph theory, coincidence degree theory and constructing suitable Lyapunov functionals.
2. The conditions we obtain can be easily checked.
3. The method of this paper shows that coupled systems on networks are easy to be studied with the aid of graph theory.

The paper is organized as follows: In Section 2, some preliminary results and our mathematical model are introduced. In Section 3, sufficient conditions for the existence of periodic solutions are obtained. Then in Section 4, we derive that the periodic solution is globally exponentially stable. Finally, a numerical simulation is carried out to illustrate the results.

2. Preliminary

In this section, we firstly introduce some necessary basic concepts and lemmas on graph theory and coincidence degree theory, which will be used in our proof. Then we describe the model in the end.

Graph theory. The following concepts on the graph theory can be found in [26]. A digraph $\mathcal{G}=(U,E)$ contains a set $U=\{1,2,\ldots,l\}$ of vertices and a set E of arcs (k,h) leading from initial vertex k to terminal vertex h. We call a digraph $\mathcal{G}$ weighted if each arc (h,k) is assigned a positive weight a_{kh}. Define the weight matrix $A=(a_{kh})_{l\times l}$. Here $a_{kh}>0$ if and only if $(h,k)\in E$ and 0 otherwise. The weight $W(\mathcal{G})$ of $\mathcal{G}$ is the product of the weights on all its arcs.

A directed path $\mathcal{P}$ in $\mathcal{G}$ is a subgraph with distinct vertices $\{i_1,i_2,\ldots,i_s\}$ such that its set of arcs is $\{(i_k,i_{k+1}):k=1,2,\ldots,s-1\}$. If $i_s=i_1$, we call $\mathcal{P}$ a directed cycle. A connected subgraph $\mathcal{T}$ is a tree if it contains no cycles. A tree $\mathcal{T}$ is rooted at vertex k, called the root, if k is not a terminal vertex of any arcs, and each of the remaining vertices is a terminal vertex of exactly one arc. A subgraph $\mathcal{Q}$ is unicyclic if it is a disjoint union of rooted trees whose roots form a directed cycle.

A digraph $\mathcal{G}$ is strongly connected if, for any pair of distinct vertices, there exists a directed path from one to the other. Denote the digraph with weight matrix A as $(\mathcal{G},A)$. The Laplacian matrix of $(\mathcal{G},A)$ is defined as

$$L=\begin{pmatrix}\sum_{k\neq 1}a_{1k} & -a_{12} & \cdots & -a_{1l}\\ -a_{21} & \sum_{k\neq 2}a_{2k} & \cdots & -a_{2l}\\ \vdots & \vdots & \ddots & \vdots\\ -a_{l1} & -a_{l2} & \cdots & \sum_{k\neq l}a_{lk}\end{pmatrix}.$$

Lemma 1. *[14]Assume $l\geq 2$. Let c_k denote the cofactor of the kth diagonal element of Laplacian matrix of $(\mathcal{G},A)$. Then the following identity holds:*

$$\sum_{k,h=1}^{l}c_k a_{kh}P_{kh}(y_k,y_h)=\sum_{Q\in\mathbb{Q}}W(Q)\sum_{(s,r)\in E(\mathfrak{C}_Q)}P_{rs}(y_r,y_s).$$

Here $P_{rs}(y_r,y_s)$, $1\leq r,s\leq l$, are arbitrary functions, $\mathbb{Q}$ is the set of all spanning unicyclic graphs of $(\mathcal{G},A)$, $W(Q)$ is the weight of Q, and $\mathfrak{C}_Q$ denotes the directed cycle of Q. In particular, if $(\mathcal{G},A)$ is strongly connected, then $c_k>0$ for $k=1,2,\ldots,l$.

Coincidence degree theory. We introduce some concepts and lemma concerning coincidence degree theory as follows. For more details, see [27].

Let X, Z be real Banach spaces, L: Dom$L\subset X\to Z$ be a linear mapping, and N: $X\to Z$ be a continuous mapping. If Dim Ker$L$ = Codim Im$L<\infty$, and ImL is closed in Z, then the mapping L is said to be a Fredholm mapping of index zero. If L is a Fredholm mapping of index zero, and there are continuous projectors P: $X\to X$ and Q: $Z\to Z$ such that ImP=Ker$L$ and Im L = Ker$Q$ = Im $(I-Q)$, then it follows that $L|$Dom$L\cap$Ker$P$: $(I-P)X\to$ ImL is invertible. Denoting the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N is called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N:\bar{\Omega}\to X$ is compact. Since ImQ is isomorphic to Ker$L$, there exists an isomorphism J: Im$Q\to$ Ker$L$.

Lemma 2. ***(The continuation theorem of coincidence degree)****[27] Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Suppose that the following conditions hold.*

(Y1) *For each $\lambda\in(0,1)$, $\forall x\in\partial\Omega\cap$DomL, $Lx\neq\lambda Nx$;*
(Y2) *For each $x\in\partial\Omega\cap$KerL, $QNx\neq 0$;*
(Y3) $\deg_B\{JQN,\Omega\cap\text{KerL},0\}\neq 0$, *where B denotes the Brouwer degree.*

Then the equation $Lx=Nx$ has at least one solution lying in DomL $\cap\,\bar{\Omega}$.

Model formulation. Given a digraph $\mathcal{G}$ with l $(l\geq 2)$ vertices, the coupled system can be built on $\mathcal{G}$ by the following process. Suppose that each vertex system dynamics is given by the following neutral differential equation with delay:

$$(x_k(t)-\gamma x_k(t-\tau_k))'=f_k(t,x_k(t),x_k(t-\tau_k)),\quad 1\leq k\leq l,$$

where $x_k(t)=(x_{k1}(t),x_{k2}(t),\ldots,x_{km}(t))^T$, τ_k denotes the time delay in the kth subsystem, $f_k(t,x,y):\mathbb{R}\times\mathbb{R}^m\times\mathbb{R}^m\to\mathbb{R}^m$ is continuous, and there is some $T>0$ such that $f_k(t,x,y)=f_k(t+T,x,y)$. If we assume that the influence of the hth subsystem on the kth subsystem is described by $H_{kh}(x_h(t)-\gamma x_h(t-\tau_h))$, $H_{kh}(0)=0$, then we can obtain a kind of NCSNDs described by