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A generalized smoothing Newton method for the symmetric cone complementarity problem ☆

Yuan-Min Li*, Deyun Wei

School of Mathematics and Statistics, Xidian University, Xi'an 710071, PR China

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ABSTRACT

In this paper, a concept of regulation functions is proposed, and some related properties and examples are explored. Based on this regulation function and some smoothing complementarity functions, we present a family of smoothing Newton methods to solve the symmetric cone complementarity problem. This algorithm allows a unified convergence analysis for some smoothing Newton methods. We show that the resulting Newton equation is well-defined and solvable, and provides a theory of global convergence.

1. Introduction

Suppose V is a Euclidean Jordan algebra, K is the corresponding cone of squares and $F: V \to V$ is a continuously differentiable mapping. The symmetric cone complementarity problem (SCCP) is to find $x \in V$ such that

$$x \in K, \ y = F(x) \in K, \ \langle x, y \rangle = 0. \tag{1}$$

This model provides a simple unified framework for various existing complementarity problems such as the standard complementarity problem over nonnegative orthant cone (NCP), the second-order cone complementarity problem (SOCCP) and the semidefinite complementarity problem (SDCP). In addition, the model itself is closely related to the KKT optimality conditions for the convex symmetric cone program (CSCP). Therefore, the SCCP has wide applications in engineering, economics, management science and other fields; see [1–4] and references therein.

Recently, there is much interest in studying optimization problems over symmetric cones. Many Newton-type methods have been proposed for solving mathematical programming over symmetric cones. For example, Yoshise introduced interior-point method to solve the monotone nonlinear SCCP [5,6]. Based on eligible kernel functions, Wang et al. extended the interior-point method to the Cartesian $P_*(k)$ linear SCCP, which is a generalization of the monotone linear SCCP [7]. The authors further presented a class of polynomial interior point algorithms using a parametric kernel function [8]. By using Jordan algebra technique, the iteration bounds were derived that match the currently best known iteration bounds for large- and small-update methods. In [9], the feasible interior-point method is based on the classical logarithmic barrier function where the iteration complexity can match the currently best known iteration bound for interior point method solving the Cartesian $P_*(k)$ linear SCCP. Besides, Lu and Huang extended the continuation method to solve the monotone SCCP[10]. Huang gave a smoothing Newton method frame

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* Corresponding author. Tel.: +8618202971504.
E-mail addresses: ymli@xidian.edu.cn, yuanminlihit@aliyun.com (Y.-M. Li), dywei@xidian.edu.cn (D. Wei).

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to solve the SCCP [11]. Based on different complementarity functions, various smoothing methods were also developed [12–22]. Among these methods, the smoothing Newton method has been paid much more attention. One kind of smoothing method is to solve a sequence of Eq. [23]

$$\phi_\mu(x) = 0, \;\; \mu > 0$$

for each μ and gradually force μ to zero. Here $\phi_\mu(x) = \phi_\mu(x, F(x))$ is a smooth approximation to a symmetric cone complementarity function ϕ and $\phi_\mu \to \phi$ as $\mu \to 0$. μ is called the smoothing parameter. At each iteration, Δx^k is a solution of the following Newton equation:

$$\phi'_\mu(x^k)\Delta x = -\phi(x^k).$$

Then $x^{k+1} = x^k + t^k \Delta x^k$, where t^k is the step size, and μ^k is updated by some specified procedures. Another kind of smoothing method is to take the smoothing parameter μ as an independent variable, i.e., μ plays the same role as the main variable x. One example of this kind of smoothing equation is [11,14,16,17,19]

$$\begin{pmatrix} \mu \\ \phi(\mu, x, y) \\ y - F(x) \end{pmatrix} = 0. \tag{2}$$

Another example is [12]

$$\begin{pmatrix} e^\mu - 1 \\ \phi(\mu, x, y) \\ y - F(x) \end{pmatrix} = 0, \tag{3}$$

where $\phi(\mu, x, y)$ is one of the smoothed Fisher-Burmeister complementarity function or the smoothed natural residual function.

Both of the smoothing Eqs. (2) and (3) can approximate the solution of the original problem. The only drawback is that the two different smoothing equations lead to different convergence analysis. In order to unify the convergence analysis, we propose the concept of regulation functions which is very useful to establish the equivalent versions of the original complementarity problem. Besides the functions μ and $e^\mu - 1$, there are many choices to be the first component function of (2) and (3) such as $\mu^2 + 3\mu$, $(\mu + 1)\ln(\mu + 1)$. In order to explore many more regulation functions, we investigate some related properties of the regulation function. Based on the regulation function and complementarity function, we transform the original SCCP (1) into the following equation system:

$$H(\mu, x, y) = \begin{pmatrix} h(\mu) \\ \phi(\mu, x, y) \\ y - F(x) \end{pmatrix} = 0, \tag{4}$$

where $h(\mu)$ is a regulation function and $\phi(\mu, x, y)$ is a smoothing complementarity function. Then we present a specific smoothing Newton method to solve the equation system (4). It is shown that this method is well-defined and the solution of (1) can be obtained from any accumulation point of the iteration sequence generated by this method. The algorithm is locally quadratically convergent under suitable conditions. Numerical results are reported for second-order cone complementarity problems, which indicate that the algorithm can solve the problems efficiently. Besides the classical regulation functions μ and $e^\mu - 1$, some other regulation functions can also be competitive, such as $(\mu + 1)\ln(\mu + 1)$, $30^\mu - 1$, and so on.

The paper is organized as follows. In the next section, we give a brief introduction to Euclidean Jordan algebra and the smoothing complementarity functions. In Section 3, we introduce the concept of regulation functions and explore some related properties and examples. In Section 4, a smoothing Newton method for SCCPs is presented. Preliminary numerical results are reported in Section 5.

2. Preliminaries

We review some basic concepts and properties which will be used in the subsequent analysis. For more details of Jordan algebras, the reader is referred to [24,25]. In addition, we recall the smoothing complementarity functions which appears in [19].

2.1. Euclidean Jordan algebra

Let V be a finite dimensional vector space over the field of real numbers. Then $(V, \circ)$ is called a Jordan algebra if a bilinear mapping $V \times V \to V$ denoted by $\circ$ is defined which satisfies $x \circ y = y \circ x$ and $L_x L_{x^2} = L_{x^2} L_x$ for any $x, y \in V$, where $x^2 = x \circ x$, L_x: $V \to V$ is a linear transformation defined by $L_x y = x \circ y$. A Jordan algebra $(V, \circ)$ is called Euclidean if an associative inner product "$\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$" is defined, i.e., $\langle x \circ y, z \rangle = \langle x, y \circ z \rangle$ holds for any $x, y, z \in V$. Also, Jordan algebras are usually defined to be unitary, that is, they contain a unit element e such that $x \circ e = e \circ x = x$ holds for all $x \in V$.

The set of squares $K = \{x^2 : x \in V\}$ is a symmetric cone of Euclidean Jordan algebra $(V, \circ, \langle \cdot,\cdot \rangle)$. This means that K is a self-dual closed convex cone and for any two elements $x, y \in \mathrm{int}(K)$, there exists an invertible linear transformation Γ: $V \to V$ such that $\Gamma(K) = K$ and $\Gamma(x) = y$. A cone is symmetric if and only if it is the cone of squares of some Euclidean Jordan algebra [24]. Thus K induces a (partial) order on V: $x \succcurlyeq y \; (y \preccurlyeq x) \Leftrightarrow x - y \in K$. We use this notation $x \prec y \; (y \succ x)$ when $y - x \in \mathrm{int}(K)$.