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A bivariate rational cubic interpolating spline with biquadratic denominator

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ABSTRACT

A bivariate rational bicubic interpolating spline (BRIS) with biquadratic denominator and six shape parameters is constructed in a rectangle domain. The C^1 continuous condition of BRIS discussed. BRIS is proved to be bounded and its error is estimated. In the case of the equally spaced knots, the matrix expression and symmetry of BRIS are presented. Some properties of the basis of BRIS are given. In the end, a numerical example is given to illustrat the effect of the shape parameters on the shape of BRIS surface.

1. Introduction

The construction method of curves and surfaces and their mathematical description is a key issue in CAGD. There are many ways [5–7,9–11,19,22,24,25,27–29] to deal with this problem, such as the polynomial spline method, Bézier spline method and NURBS method. However, most of the polynomial spline methods are the interpolating cases, and their local shapes can not be modified for the interpolating surfaces while interpolating data is unchanged. NURBS and Bézier methods are the non-interpolating cases, that is to say, the constructed curve and surface do not fit with the given data and the given points are the control points. Therefore, when we construct the interpolating functions required for CAGD, we should consider the following cases: 1) The expressions of interpolating functions are simple and explicit. 2) The parameters of constructed curves and surfaces can be modified without changing the given data.

Recently, there has been many works [3,4,8,12,18,20,21,26] about univariate rational spline interpolation with parameters. Some univariate rational interpolating splines are generalized to bivariate rational interpolating splines which expressions with parameters are simple and explicit. Some interesting results are presented. In [2,13–17,23,30], the authors constructed several bivariate interpolating splines over rectangular mesh, derived some properties such as the sufficient conditions of down-constrained and up-constrained for the shape control, the matrix expression, bounded property, stability, convexity control, the preserving positivity.

In this paper, motivated by [2,15], we generate the bivariate rational interpolating spline with four shape parameters in [15] to the case with six shape parameters (BRIS). We present the definition of BRIS and discuss the effect of the parameter choice on the curve shape in Section 2. We give the sufficient condition that BRIS is C^1 continuous in Section 3. We prove the bounded property and analyse the error of BRIS in Section 4. In the case of the equally spaced knots, we present the matrix expression of

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Table 1
Data points from [1].

i	1	2	3	4	5	6	7	8	9
x_i	0	0.25	0.5	1	1.5	2	2.5	3	4
f_{ij}	2	0.6	0.1	0.13	1	0.5	1.1	0.25	0.2

BRIS, prove that it is symmetry, and discuss some properties of the basis of BRIS in Section 5. We give a numerical example of BRIS to illustrat the effect of the shape parameters on the shape of BRIS surface in Section 6.

2. Rational interpolating spline

Let D be the rectangular domain $[a, b, c, d]$, $\{(x_i, y_i), f_{ij}, i = 1, 2, \dots, n+1; j = 1, 2, \dots, m+1\}$ be a given set of data points, where $a = x_1 < x_2 < \cdots < x_{n+1} = b$; $c = y_1 < y_2 < \cdots < y_{m+1} = d$. Set

$$f_{ij} = f(x_i, y_j), h_i = x_{i+1} - x_i, l_j = y_{j+1} - y_j.$$

Denote $D_{ij} = [x_i, x_{i+1} y_j, y_{j+1}], i = 1, 2, \dots, n, j = 1, 2, \dots, m$. For any point $(x, y) \in D_{ij}$, let

$$\theta = (x - x_i)/h_i, \eta = (y - y_j)/l_j.$$

For each $y_j, j = 1, 2, \dots, m+1$, we construct the x-direction interpolation curve as follows

$$P^*_{ij}(x) = \frac{p^*_{ij}(x)}{q^*_{ij}(x)}, i = 1, 2, \dots, n-1, \tag{1}$$

where

$$p^*_{ij}(x) = \alpha^*_{ij} f_{ij}(1-\theta)^3 + V^*_{ij}\theta(1-\theta)^2 + W^*_{ij}\theta^2(1-\theta) + \gamma^*_{ij} f_{i+1,j}\theta^3$$
$$q^*_{ij}(x) = \alpha^*_{ij}(1-\theta)^2 + \beta^*_{ij}\theta(1-\theta) + \gamma^*_{ij}\theta^2$$

and

$$V^*_{ij} = \beta^*_{ij} f_{ij} + \alpha^*_{ij} f_{i+1,j}$$
$$W^*_{ij} = (\beta^*_{ij} + \gamma^*_{ij}) f_{i+1,j} - \gamma^*_{ij} h_i \Delta^*_{i+1}$$

with $\alpha^*_{ij}, \beta^*_{ij}, \gamma^*_{ij}$ positive, and $\Delta^*_i = \frac{f_{i+1,j} - f_{ij}}{h_i}$. We can prove that

$$P^*_{ij}(x_i) = f_{ij}, P^*_{ij}(x_{i+1}) = f_{i+1,j}, P^{*\prime}_{ij}(x_i) = \Delta^*_{ij}, P^{*\prime}_{ij}(x_{i+1}) = \Delta^*_{i+1,j}.$$

The interpolation (1) is called the rational cubic interpolating spline. It is clear that the interpolation is local in the interval $[x_i, x_{i+1}]$ and depends on the data at three points $\{(x_r, y_j), f_{rj}\}, r = i, i+1, i+2$ and the shape parameters $\alpha^*_{ij}, \beta^*_{ij}, \gamma^*_{ij}$.

It is interesting to note that the interpolation (1) becomes a standard cubic Hermite spline with the values of the shape parameters $\alpha^*_{ij} = 1, \beta^*_{ij} = 2, \gamma^*_{ij} = 1$. we now illustrate the mathematical and graphical effects of the shape parameters $\alpha^*_{ij}, \beta^*_{ij}, \gamma^*_{ij}$ on the shape of a curve. The three free parameters can be exploited properly to modify the shape of curve according to the designer's choice. We rewrite (1) as follows:

$$P^*_{ij}(x) = \frac{\alpha^*_{ij}(1-\theta)^2(f_{ij}(1-\theta) + f_{i+1,j}\theta) + \beta^*_{ij}\theta(1-\theta)(f_{ij}(1-\theta) + f_{i+1,j}\theta) + \gamma^*_{ij}\theta^2(f_{i+1,j} - h_i(1-\theta)\Delta_{i+1,j})}{\alpha^*_{ij}(1-\theta)^2 + \beta^*_{ij}\theta(1-\theta) + \gamma^*_{ij}\theta^2} \tag{2}$$

By the computation for Eq. (2) we have the following formulas:

$$\lim_{\alpha^*_{ij}\to\infty} P^*_{ij}(x) = f_{ij}(1-\theta) + f_{i+1,j}\theta \tag{3}$$

$$\lim_{\beta^*_{ij}\to\infty} P^*_{ij}(x) = f_{ij}(1-\theta) + f_{i+1,j}\theta \tag{4}$$

$$\lim_{\gamma^*_{ij}\to\infty} P^*_{ij}(x) = f_{i+1,j} - (1-\theta)h_i\Delta_{i+1,j} \tag{5}$$

We can see from Eqs. (3) and (4) that the increase in the shape parameter α^*_{ij} or β^*_{ij} reduces the rational interpolating spline (2) to the straight line $f_{ij}(1-\theta) + f_{i+1,j}\theta$, and see from Eq. (5) that the increase in γ^*_{ij} reduces (2) to another straight line $f_{i+1,j} - (1-\theta)h_i\Delta_{i+1,j}$. According to data points in Table 1 from [1], by choosing different shape parameters, we can observe the corresponding shape changes of the interpolating curves in Figs. 1–4. Each piecewise in Fig. 1 slopes heavily at the left-hand side. Each piecewise in Fig. 2 and 3 incline to a line segment. The curve in Fig. 4 is a cubic Hermite interpolating spline. We remark that in order to plot the 8th interpolating spline curve, we add the point $\{x_{10}, f_{10}\} = \{4.5, 0.2\}$.