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The MGPBiCG method for solving the generalized coupled Sylvester-conjugate matrix equations ☆

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ABSTRACT

In this paper, we extend the generalized product-type bi-conjugate gradient (GPBiCG) method for solving the generalized Sylvester-conjugate matrix equations $A_1XB_1 + C_1\overline{Y}D_1 = S_1, A_2\overline{X}B_2 + C_2YD_2 = S_2$ by the real representation of the complex matrix and the properties of Kronecker product and vectorization operator. Some numerical experiments demonstrate that the introduced iteration approach is efficient.

1. Introduction

We consider the Sylvester matrix equations of the form

$$\begin{cases} A_1XB_1 + C_1\overline{Y}D_1 = S_1, \\ A_2\overline{X}B_2 + C_2YD_2 = S_2, \end{cases} \tag{1.1}$$

where $A_i, B_i, C_i, D_i\ (i = 1, 2), S_1, S_2 \in C^{n\times n}$ are given matrices and $X, Y \in C^{n\times n}$ are the matrices to be determined.

The Sylvester matrix equations have numerous applications in control theory, model reduction, system stability, image restoration, pole assignment, observer design, filtering, etc. [1–10]. Researches on different kind of the Sylvester matrix equations appear in many papers. Xie et al. proposed the MCG method for solving the generalized coupled Sylvester-transpose linear matrix equations $AXB + CY^TD = S_1$, $EX^TF + GYH = S_2$ over the reflexive and anti-reflexive solutions [11]. Liang et al. presented a modified conjugate gradient method for solving the equations $A_1XB_1 + C_1X^TD_1 = F_1$, $A_2XB_2 + C_2X^TD_2 = F_2$ [12]. Ding et al. gave the iterative solutions of the generalized Sylvester matrix equations by using the hierarchical identification principle [13]. Liao et al. obtained the least squares solution with the minimum-norm for the matrix equations $AXB = C$, $GXH = D$ in [14]. Lee et al. considered a sufficient and necessary solvability condition for the mixed generalized Sylvester matrix equations $A_1X - YB_1 = C_1$, $A_2Z - YB_2 = C_2$ [15], moreover, Wang et al. provided a new sufficient and necessary solvability condition for the same system

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and gave a general solution expression as the system is solvable [16]. Dehghan et al. investigated some iterative methods for solving generalized Sylvester matrix equations with the generalized bisymmetric and skew-symmetric matrices in [17]. An iterative approach was derived by Hajarian in [18] for a periodic Sylvester matrix equations $\widehat{A}_j\widehat{X}_j\widehat{B}_j + \widehat{C}_j\widehat{X}_{j+1}\widehat{D}_j = \widehat{E}_j$ with the period solutions $\widehat{X}_{j+\lambda} = \widehat{X}_j$, where λ denotes the period, $(j = 1, 2,\ldots)$. In [19], the author extended the GPBiCG algorithm for solving a generalized Sylvester-transpose matrix equation $\sum_{j=1}^{r}(A_iX_iB_i + C_iX^TD_i) = E$.

Some complex matrix equations have attracted much attention from many researchers, such as in [20] which shows that the consistence of the Sylvester-conjugate matrix equation $AX - \overline{X}B = C$ related to the consimilarity of two matrices [21–23]. By means of a real representation of a complex matrix, the Kalman–Yakubovich conjugate matrix equation $X - A\overline{X}B = C$ was investigated in [24]. Xie et al. constructed an efficient algorithm to solve the centrally symmetric (centrally anti–symmetric) solution of the generalized coupled Sylvester-conjugate matrix equations $A_1X + B_1Y = D_1\overline{X}E_1 + F_1$, $A_2Y + B_2X = D_2\overline{Y}E_2 + F_2$ in [25]. Based on the properties of the controllable and observable matrices, Wu et al. obtained the expression of the exact solution for the matrix equations $V - AVF = BW$ and $V - A\overline{V}F = BW$ [26,27]. Furthermore, they also considered the iterative approach by using the hierarchical identification principle for solving the Sylvester matrix

$$\sum_{\eta=1}^{p}(A_{i\eta}X_\eta B_{i\eta} + C_{i\eta}\overline{X}_\eta D_{i\eta}) = F_i,\ \ i \in I(1, N),$$

where $I[1, N]$ denotes the set $\{1, 2, , N\}$ [28]. On the basis of [28], Song et al. presented some iterative methods for solving the matrix equations

$$\sum_{\eta=1}^{r}A_iX_iB_i + \sum_{j=1}^{s}C_jX_i^HD_j = E,\ \ i \in I(1, N)$$

and

$$\sum_{\eta=1}^{p}\left(A_{i\eta}X_\eta B_{i\eta} + C_{i\eta}X_\eta^TD_{i\eta}\right) = F_i,\ \ i \in I(1, N),$$

where $A_{i\eta} \in R^{m_i\times l_\eta}$, $B_{i\eta} \in R^{n_\eta\times p_i}$, $C_{i\eta} \in R^{m_i\times n_\eta}$, $D_{i\eta} \in R^{l_\eta\times p_i}$, $F_{i\eta} \in R^{m_i\times n_i}$ are given matrices, $X_\eta \in R^{l_\eta\times n_\eta}$ are the matrices to be determined [29,30].

As known, the generalized product-type bi-Conjugate Gradient (GPBiCG) method is regarded as an efficient approach for solving the nonsymmetric linear system

$$Ax = f, \tag{1.2}$$

where $A \in R^{m\times m}, f \in R^m$. Inspired by the [19,31], in this work, we extend the GPBiCG method to get the matrix iterative scheme for solving the generalized coupled Sylvester-conjugate matrix equations $A_1XB_1 + C_1\overline{Y}D_1 = E$, $A_2\overline{X}B_2 + C_2YD_2 = F$ by the real representation of the complex matrix and the properties of Kronecker product and vectorization operator. In real case, if A_2, B_2, C_2, D_2 are zero, then (1.1) becomes the problem discussed by Liao, et al. in [32]. Further special cases can be found also in Liao et al.'s papers, where the solvability, solution formula and factorization algorithms are studied [33–38].

For the convenience of our statements, we use the following notation throughout the paper: Let $R^{m\times n}$ and $C^{m\times n}$ denote the set of $m \times n$ real matrix and $m \times n$ complex matrix, respectively. For $x \in R^n$, $\|x\|$ denotes the Euclidean norm. For $A \in C^{m\times n}$, we write $\mathrm{Re}(A)$, $\mathrm{Im}(A)$, $\overline{A}$, A^T, A^H, $\|A\|_F$ to denote the real part, the imaginary part, the conjugation, transpose, conjugate transpose, the Frobenius norm, respectively. The matrix $\mathrm{Diag}\{A_1, A_2,\ldots, A_n\}$ denotes the block diagonal matrix with $A_i \in R^{m\times m}$ $(i = 1, 2,\ldots, n)$. For any $A = (a_{ij})$, $B = (b_{ij})$, $A\otimes B$ denotes the Kronecker product defined as $A\otimes B = (a_{ij}B)$, $i = 1, 2,\ldots, m, j = 1, 2,\ldots, n$. For the matrix $X = (x_1, x_2,\ldots, x_n) \in C^{m\times n}$, $\mathrm{vec}(X)$ denotes the vectorization operator defined as $\mathrm{vec}(X) = (x_1^T, x_2^T, \ldots, x_n^T)^T \in C^{mn}$. The inner product in space $C^{m\times n}$ is defined as

$$\langle A, B\rangle = \mathrm{Re}[\mathrm{tr}(A^HB)]. \tag{1.3}$$

I denotes the unit matrix of the appropriate dimension. i denotes the imaginary unit.

The remainder of the paper is organized as follows. In Section 2, we review the GPBiCG method which is a powerful approach for solving the nonsymmetric linear system . In Section 3, we generalize the method for solving the matrix Eq. (1.1) by the real representation of the complex matrix and the properties of Kronecker product and vectorization operator. Some numerical experiments are given in Section 4, which illustrate that the extending approach is efficient. At last, we end the paper with some conclusions in Section 5.

2. The GPBiCG methods

Firstly, we will briefly recall the basic idea and the principle of the GPBiCG method in [31], then show the method for solving the nonsymmetric linear system (1.2).

The conjugate gradient (CG) method is an efficient method for solving the large linear system. The bi-conjugate gradient (Bi-CG) method, a.k.a. Petrov–Galerkin method, is a popular method for solving the large sparse nonsymmetric linear system. However, the Bi-CG method tends to numerical instabilities even breakdowns in the iterative process. The conjugate gradient