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Stability analysis of turbulent heat exchange over the heterogeneous environmental interface in climate models

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ABSTRACT

Based on a short review of the different parameterization schemes for sub-grid scale surface fluxes in climate and other atmospheric models of different scales, the flux aggregation effect over a heterogeneous grid-box leading to the occurrence of Schmidt's paradox is considered. To investigate this effect in the sub-grid scale parameterization, we have introduced a dynamical system approach, where the horizontal energy exchange is taken into account and is represented by a matrix of coupling parameters. Since it is, in general, very difficult to specify the quantities in that matrix, a sufficient condition for the asymptotic stability that can be applied for any coupling matrix is derived. Two theorems that consider the flux aggregation effect over a heterogeneous grid-box are proved. Finally, we have showed how, by their application, Schmidt's paradox can be overcome. It is demonstrated through a numerical example of turbulent energy exchange over the grid-box including the part of the Prospect park, New York, USA.

1. Introduction

1.1. General

The Earth's climate system, joining the physical and chemical components of the atmosphere, ocean, land surface, and cryosphere is the target of global climate models whose objective is to correctly simulate the spatial variation of climate in some average sense. The main current issues in modeling the global climate system can be summarized as follows [4]. (i) Chaos. This is a deterministic chaos whose source are nonlinearities in the Navier–Stokes equations and its sensitivity to initial conditions. However, in addition, in climate models, coupling of a nonlinear model over one environmental interface to a nonlinear model over another environmental interface (for example, land and ocean), gives rise to something much more complex than the deterministic chaos of the weather model, leading to bifurcation, instability, and chaos [1]. (ii) Confidence in climate models. This issue is how well the climate model reproduces reality, that is, whether the model works and is it fit for its intended purpose. (iii) Climate model imperfection. The meaning of this issue can be addressed to the fact that our understanding of, and ability to simulate, the Earth's climate is rather limited. The climate model imperfection is divided into two types: uncertainty

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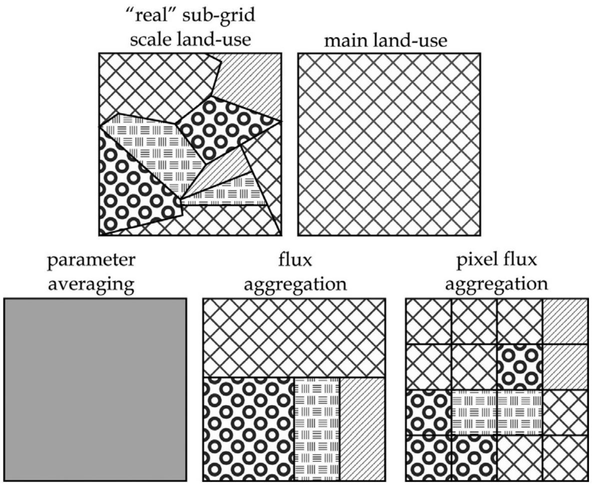

Fig. 1. Schematic diagram illustrating how the sub-grid scale surface patch-use classes are treated within the different parameterization schemes for sub-grid scale surface fluxes in climate and other models of different scales.

and inadequacy. The term model uncertainty means that we cannot reliably choose parameters, which will give the most informative results; the term inadequacy means that before we run any simulation of the future, we know in advance that models are not realistic representations of many key aspects of the real system [18]. (iv) Sub-grid scale parameterization. Except uncertainty in model parameters and initial conditions, the model uncertainty is associated with sub-grid scale parameterizations (e.g. boundary layer turbulence, cloud microphysics) generating systematic errors in meteorological fields which are obtained by the downscaling procedure.

1.2. The sub-grid scale parameterization of turbulent sensible heat flux in climate models

The approaches for calculating the turbulent transfer of momentum, heat and moisture from a grid-box composed of heterogeneous surfaces to the atmosphere can be classified as follows (Fig. 1). (a) Main patch-use. This is the simplest method to yield a grid box representative surface flux over heterogeneous terrain. In this approach surface fluxes are calculated based on the sub-grid scale patch-use class characteristics that make up the largest fraction of the grid-box. Its advantage is that it is computationally cheap but in very heterogeneous areas this approach is not recommended. (2) Parameter averaging, where grid-box mean radiation, aerodynamic, physiological, morphological and soil parameters are derived to best incorporate the combined nonlinear effects of each of the different relatively homogeneous sub-regions (patches) over the grid-box. Note that this approach, which is the favorable one concerning computing time, can produce even worse results than the main land-use approach especially for very coarse resolutions. (3) Flux aggregation, is the method where the fluxes are averaged over the grid-box. (4) Pixel flux aggregation. This mosaic flux aggregation method is optimal for future pixel-based patch-use data sets, although it is computationally relatively expensive [3]. (5) Combined method is a combination of (2) and (3) methods [6].

1.3. Focus of the paper

When either flux aggregation method is used or the methods of parameter aggregation and flux aggregation are combined then we have occurrence of Schmidt's paradox [10,12]. This paradox describes situation when small regions of pronounced surface heterogeneity, with intense upward-directed turbulent sensible heat fluxes can dominate the grid-area averaged value of these fluxes, while the mean gradient of potential temperature still indicates an overall stable stratification between the surface and the lowest climate model level. Further, this situation causes arising of counter-gradient heat transfer. Thus, the sub-grid scale surface flux parameterization have to capture this phenomenon in order to derive a representative grid-box averaged flux and further yield the correct mean temperature gradient by allowing a transport of heat counter the mean gradient. Whether or not this kind of transfer leads to numerical problems depends on how the numerics are done [6]. Thus, Lamb and Durran in [9] pointed out that parabolic differential equations like the diffusion equation lead to singular solutions, if a negative eddy diffusivity is used as an option. Actually, in climate models the negative diffusivity is not used, but only implied from diagnostic grid-box averaged surface values what provides a stability of the numerics. However, regardless of which approach is applied, the physics of the counter-gradient relationship must still be accounted for.

The objectives of this paper are (i) To consider the instability that holds from the sub-grid scale parameterization of turbulent heat exchange between coupled environmental interfaces in modeling the global climate system, where the term environmental interface is defined as interface between two biotic or abiotic environments that are in relative motion and exchange energy, matter, and information through physical, biological, and chemical processes, fluctuating temporally and spatially regardless of space and time scale [14]. (ii) To investigate the problem of the aggregation in the sub-grid scale parameterization, we have