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Numeric-analytic solutions of mixed-type systems of balance laws

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ABSTRACT

The aim of the present analysis is to apply two relatively recent methods, reduced differential transform method (RDTM) and differential transform method (DTM), for the solution of balance law systems. New generalized transformed formulas are derived. The new approaches provided the solution in the form of a rapidly convergent series with easily computable components in the RDTM case, and costly components for the DTM. A comparison between the two methodologies shows that the RDTM is more effective, efficient, powerful and able to be applicable for large class of nonlinear partial differential equations than the DTM. Two test modeling problems are discussed to illustrate the effectiveness and performance of RDTM.

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1. Introduction

Balance laws are first order quasi-linear PDEs that arise in many physical applications. They assert that the change of the total amount of substance contained in a fixed domain $\Omega \subset \mathbb{R}^2$ is equal to the flux of that substance across the boundary of Ω plus the rate that the substance created in Ω. The two dimensional system of balance laws has the form

$$\begin{array}{l} \frac{\partial}{\partial t}u(x,t) + \frac{\partial}{\partial x}f(x,t,u,v) = h_1(x,t) \\ \frac{\partial}{\partial t}v(x,t) + \frac{\partial}{\partial x}g(x,t,u,v) = h_2(x,t) \end{array}, \quad (x,t) \in \Omega \subset \mathbb{R}^2, \tag{1}$$

subject to initial data

$$\begin{array}{l} u(x,0) = u_0(x) \\ v(x,0) = v_0(x) \end{array}, \tag{2}$$

The system in Eq. (1) is said to be hyperbolic in a domain $\Omega \subset \mathbb{R}^2$ if for every x, t and all possible solutions u and v of the system in Ω, the Jacobian matrix

$$\begin{pmatrix} \partial f/\partial u & \partial f/\partial v \\ \partial g/\partial u & \partial g/\partial v \end{pmatrix} = \begin{pmatrix} f_u & f_v \\ g_u & g_v \end{pmatrix} \tag{3}$$

has two real distinct eigenvalues $\lambda_1(x,t,u,v)$ and $\lambda_2(x,t,u,v)$. If there is a region $E \subset \mathbb{R}^2$ where the Jacobian matrix has no real eigenvalues the system is called elliptic. If neither Ω nor E is empty, then the system is of mixed type. In this work, we limit our study to the mixed type two dimensional system of balance laws.

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During the past three decades, many methods were developed for solving ordinary, partial and fractional differential equations and their systems. The homotopy analysis method (HAM) [1–4], the homotopy perturbation method (HPM) [5–8], the variational iteration method (VIM) [9–12], and the Adomian decomposition method (ADM) [13–16] are some examples of these methods.

The basic motivation of present study is the extension of a two developed techniques, the differential transform method (DTM) and the reduced differential transform method (RDTM), to tackle system of balance laws in Eq. (1). These semi-analytic methods are based on Taylor's theorem which requires symbolic computation of the necessary derivatives for data functions.

2. The methods

2.1. Two-dimensional differential transform method

The DTM was first applied by Pukhov [17] and Zhou [18] to solve linear and nonlinear initial value problems in electric circuit analysis. It is based on Taylor expansion series. But, it differs from the high order Taylor series by the way of calculating coefficients. The DTM has been extensively used to solve various linear and nonlinear ordinary, partial, fractional and integro-differential equations and their coupled versions [19–21]. See also [22] and references therein.

With reference to the works of Ayaz [19,20], the basic definitions and theorems of the two-dimensional DTM are introduced as follows:

Consider a function of two variables $u(x,t)$ and suppose it can be represented as a product of two single-variable functions $f(x)g(t)$. On the basis of properties of the one-dimensional differential transform, the function $u(x,t)$ can be represented as

$$u(x,t)=\sum_{i=0}^{\infty}F(i)x^i\sum_{j=0}^{\infty}G(j)t^j=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}U(i,j)x^it^j, \tag{4}$$

where, $U(i,j)=F(i)G(j)$ is called the spectrum of $u(x,t)$.

Definition 2.1.1. If the function $u(x,t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let,

$$U(k,h)=\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k\partial t^h}u(x,t)\right]_{x=t=0}, \tag{5}$$

where, the spectrum function $U(k,h)$ is the transformed function. In brief, it is called T–function.

In this work, the lowercase $u(x,t)$ represent the original function while the uppercase $U(k,h)$ stands for the transformed function.

Definition 2.1.2. The differential inverse transform of $U(k,h)$ is defined as follows:

$$u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)x^kt^h. \tag{6}$$

Combining Eqs. (5) and (6) yields

$$u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k\partial t^h}u(x,t)\right]_{(0,0)}x^kt^h. \tag{7}$$

The fundamental theorems of DTM [19,20] are:

Theorem 2.1.1. *If $u(x,t)=v(x,t)\pm w(x,t)$ then,*

$$U(k,h)=V(k,h)\pm W(k,h). \tag{8}$$

Theorem 2.1.2. *If $u(x,t)=\alpha\, v(x,t)$ then,*

$$U(k,h)=\alpha\, V(k,h). \tag{9}$$

where α is an arbitrary constant.

Theorem 2.1.3. *If $u(x,t)=x^mt^n$ then,*

$$U(k,h)=\delta(k-m,h-n)=\begin{cases}1, & k=m,\quad h=n\\ 0, & otherwise\quad s\end{cases}. \tag{10}$$

Theorem 2.1.4. *If $u(x,t)=v(x,t)w(x,t)$ then,*

$$U(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}V(r,h-s)W(k-r,s). \tag{11}$$

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