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Energy-based formulation for nonlinear normal modes in cable-stayed beam

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ABSTRACT

Based on Hamilton's variational principle, the governing equations for in-plane dynamics of the model are obtained. Nonlinear normal modes for composite structure cable-stayed beam have been studied extensively in the literature. When all particles of the system reach their extremum values at the same instant of time, there are free periodic motions. A partial differential equation related to the modal function has been constructed by means of conservation of energy, which can be solved using a perturbation methodology. Most studies have been limited to uncoupling nonlinear terms of the system. This work investigated the nonlinear normal modes in the system that contains coupling nonlinear terms. The results of the two classes of nonlinear terms are also compared in this study.

1. Introduction

The theory of linear oscillatory systems shows that the principle of linear superposition does not apply in nonlinear systems. Thus, the concept of normal mode is extended to nonlinear theory. The concept of nonlinear normal modes (NNMs) of vibratory systems can be regarded as an extension of linear normal modes. Based on the concept of invariant manifolds, the concept of nonlinear normal modes (NNMs) was originally proposed by Rosenberg [1,2]. Whereafter, many researchers have made important contributions to the problem of defining theoretically and constructing analytically the nonlinear normal modes [3]. Since then, a large body of literature has addressed, with notable success, the qualitative and quantitative analysis of nonlinear phenomena using NNMs [4]. NNMs are a useful mathematical tool to obtain minimal descriptions in structural dynamics and also to identify the underlying structure of the system nonlinear response [5]. They have the potential to handle strong structural nonlinearity and address the individualistic nature of nonlinear systems [6,7].

There are some important developments that have occurred to date in the study of NNMs [8,9]. For the discrete system, it assumes the solution as an expansion in terms of basis functions from a complete set and then uses one of the variants of the method of weighted residuals to obtain an infinite set of ordinary-differential equations [10,11]. The infinite set of equations is truncated to practically compute the nonlinear normal modes [12]. Then, the discretized equations are treated with the real-valued or complex-valued form of the invariant-manifold approach, the energy approach, or an asymptotic method [13]. For

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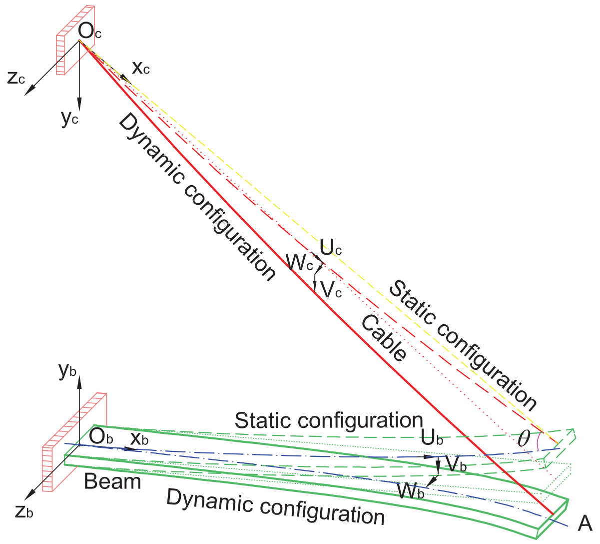

Fig. 1. Static configuration and dynamic configuration of the cable-stayed beam.

the continuous systems, some studies used direct analytical techniques, such as the method of multiple scales to construct the nonlinear normal modes [14].

The principal aim of this paper is to discussing the concept of non-linear normal mode [15]. It is attempted to show how this concept can be used to better understand the free dynamics of nonlinear oscillators [16]. It is noted that the system is conservative and the boundary conditions involving no dissipation of energy [17,18]. In this work, nonlinear normal mode of the continuous system cable-stayed beam are studied with a methodology based on the previous work. Only the displacement of the reference point is taken into account in this work, and the nonlinear normal modes are computed by imposing the condition of conservation of energy. Using a perturbation analysis, the nonlinear normal modes are approximately solved. Following, the differences between the nonlinear coupling term [19] and uncoupling term effects of the whole system are investigated.

2. Formulation of the motion

Two Cartesian coordinate systems are chosen to derive the equations of motion, as shown in Fig. 1. For the coordinate system $O_c - x_c y_c z_c$ $(O_b - x_b y_b z_b)$, the origin $O_{c(b)}$ is placed at the left support of the cable (beam). The static (dashed line) and dynamic (solid line) configurations of the cable-stayed beam are shown in Fig. 1. The three-dimensional displacements of the cable (beam) are denoted by $U_c(x_c, t)$ $(U_b(x_b, t))$, $V_c(x_c, t)$ $(V_b(x_b, t))$ and $W_c(x_c, t)$ $(W_b(x_b, t))$ along the $x_c(x_b)$, $y_c(y_b)$ and $z_c(z_b)$ directions, respectively. Moreover, we assume that the bending and torsional shear rigidities of the cable are neglected. We also neglect the torsional and shear rigidities of the beam. The symbolic meanings are provided with this paper (see Appendix).

2.1. Variational formulation

By applying means of the extended Hamilton principle, we can obtain the motion equations of the system. The extended Hamilton principle can be expressed as follows:

$$\int_{t_1}^{t_2} \delta(T - U)\mathrm{d}t + \int_{t_1}^{t_2} \delta W \mathrm{d}t = 0 \tag{1}$$

where T and U are the kinetic energy and the stain energy of the system, W is the work of non-conservative forces, δ is the sign of variation.

As aforementioned, the static configuration should be concerned due to the self-weight of the cable and beam. The static configuration of the cable can be obtained by omitting the dynamic terms. Hence, the static configuration of the cable can be written as follows:

$$y_c(x_c) = 4bx_c(l_c - x_c)/l_c^2 \tag{2}$$

where b is the sag of the cable. The axial component of the initial tension is $H_i = m_c g l_c^2/8b$. Although the contribution of the static configuration on the strain energy of the beam is negligible, the shear force of the beam produced by the static configuration is still considered in the force balance. Under the previous assumptions and by using the classical extended Hamilton principle [20], the equations of motion governing the vibration are obtained.