Applied Mathematics and Computation 265 (2015) 237–244

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Investigation of cumulative growth process via Fibonacci method and fractional calculus

F. Buyukkilic*, Z. Ok Bayrakdar, D. Demirhan

Department of Physics, Faculty of Science, Ege University, Izmir, 35100 Bornova, Turkey

ARTICLE INFO

MSC:
26A33
11B39
33E12,

Keywords:
Cumulative growth
Fractional calculus
Rate equation
Mittag–Leffler

ABSTRACT

In this study, cumulative growth of a physical quantity with Fibonacci method and fractional calculus is handled. The development of the growth process is described in terms of Fibonacci numbers, Mittag–Leffler and exponential functions. A compound growth process with the contribution of a constant quantity is also discussed. For the accumulation of residual quantity, equilibrium and lessening cases are discussed. To the best of our knowledge; compound growth process is solved for the first time in the framework of fractional calculus. In this sense, differintegral order of fractional calculus α has been achieved a physical content. It is emphasized that, in the basis of qualification of the fractional calculus for describing genuine complex physical systems with respect to ordinary descriptions is the cumulative growth mechanism with Fibonacci method. It is concluded that compound diminution and growth process mechanisms can be taken as a basis for the comprehension of derivative and integral operations in fractional calculus.

1. Introduction

In ordinary approach, it is intimately accepted that the physical phenomenon evolve in a continuous Euclidean space and time with a Markovian memory. In this standard framework, deterministic mechanic is sufficient in describing dynamical systems. But the real processes of stochastic systems evolve in a fractal medium and having discrete time intervals as well as a non-Markovian memory. In this context, however, Hamiltonian mechanics and ordinary calculus are insufficient in describing stochastic systems. In this sense, fractional calculus is very successful in describing stochastic systems [1–33]. On the other hand, the elements of fractional mathematics are given with different definitions, regardless of their applicability, consequently this situation creates difficulties for research workers. For instance, the operation of differintegral is not expressed rigorously and uniquely. This uncertainty constitutes an obstruction for those who want to use fractional calculus.

In this study, the existence of the fundamental physical mechanism on which the differintegral of fractional calculus is based on is uncovered with Fibonacci approach and compound growth process. For instance, it is concluded that from the solutions of rate equation using fractional calculus, the order α of differintegral can be interpreted as a parameter that controls the evolution of stochastic processes. The growth process that is outlined in this study develops in a fractal medium in a discrete time and space with a non-Markovian memory. Thus, a mathematical bridge is established between the present and future values of the quantity which is under investigation. In this regard one of the functions of the fractional calculus namely Mittag–Leffler (M–L)

* Corresponding author. Tel.: +90 2323112846.
E-mail address: fevzi.buyukkilic@ege.edu.tr (F. Buyukkilic).

http://dx.doi.org/10.1016/j.amc.2015.05.026

function is achieved by cumulative growth process should be noted. It is expected that, to an extent, the inadequacies of the standard approach are overcome [34–50].

In order to justify the vibrancy of field of this research, recent articles which are within the framework of social science should be mentioned. The quantitative methods of physical sciences are also applicable to social sciences. Cumulative growth processes are ubiquitous in everyday life and it is a relevant set up to study social events as well. In this content, compound growth process plays a crucial role for the following examples: Evolution of the most common English words and phrases over the centuries within this frame work [51]. The quest concerning to the development of physical discoveries across over the two centuries are given in [52]. It is reported that the emergence and diminution of scientific paradigm are governed by robust principles of self-organization. Languages can be treated as living creatures. With the use of quantitative methods of science and analyzing the usage of words in several languages, authors of Ref. [53] came to the conclusion that, languages expands and contract within allometric proportions which is communicated in their concomitant paper.

As a conclusion it is communicated that those dynamical very different entities that involve cumulative growth and cumulative diminutions processes are those events which self organize in the progress of time. Within this ensemble, following examples namely, preferential attachment in wealth, preferential attachment in network sizes, cumulative advantage, carrier longevity, corporation sizes, population of cities, avalanches, forest-fire size, efficacy in technological systems, etc., can be recited [54].

This study is arranged as follows: In Section 1, the well known rate equation is handled for a physical quantity A and the solution for growth process is investigated. It is pointed out that the solution which is done by standard calculus is carried out for a regular growth which evolves in a continuous Euclidian space and continuous time, with a Markovian memory [21,26]. In Section 2, the solution of rate equation is presented for the growth process which evolves in fractal space, with a discrete time and a non-Markovian memory, a cumulative growth by Fibonacci approach is used [34,35,38,43,44]. A relation between the present value and the future value of the quantity A is established. In Section 3, to the best of our knowledge, it is the first time that the fractional growth process is handled and solved within the framework of fractional calculus. By comparing the solution of cumulative method and the solution of fractional calculus, a physical explanation/content is given to the order of differintegral, namely α [21,22,25,26,34,35,39,42,55,59]. In Section 4, a cumulative growth process with an initial value A_0, and with a constant contribution value of P in each step is handled. It is observed that when time is fixed for the cumulative growth description of the processes, the universal Fibonacci numbers are the main elements of description.

2. Cumulative growth is revisited

The events in nature and human behaviors which are under study, compoundly evolve in a space and time. In this section, a cumulative growth process in a free medium is introduced. In nature, as the examples of the development of the complex stochastic systems, earthquakes, fire, avalanche, cancerous cells, germ colonies, meteorological events, etc. can be seen very commonly. On the other hand, as examples of the human activities; stock exchange index, interest income of the capital movements, insurance operations, telephone central, vehicle numbers in traffic, etc. exhibit compound behaviors [24,34,35,40,45–49]. For the common theory of nature and human behaviors, let the $A(t)$ be the quantity under investigation. Suggesting that the change $dA(t)$ is proportional to $A(t)$ and time interval dt. For this case

$$dA(t) = \lambda A(t)dt. \tag{1}$$

can be written, where

$$\lambda = \frac{1}{dt}\frac{dA(t)}{A(t)} \tag{2}$$

is the growth rate and has the dimension of inverse time. The sign "+" on the right hand side of the Eq. (1), points out that the physical quantity $A(t)$ is subjected to a growth. In principle, Eq. (1) can be written in the form:

$$\frac{dA(t)}{A(t)} - \lambda dt = 0. \tag{3}$$

The above equation is the well-known linear, homogeneous rate equation. The solution of the rate Eq. (1) is determined as,

$$A(z) = A(0)e^{z} \tag{4}$$

where, $A(0)$ is the initial value at $t = 0$ and

$$z = \lambda t \tag{5}$$

Naturally, the relation between the initial value $A(0)$ and the future value $A(z)$ is attained in exponential form. It is evident that Eq. (4) is far from describing realistic physical systems. Here, the existence of an external environment which speeds up and slows down the process, the fractal character of the space and the discrete character of time and the memory effects are ignored. It is seen that the process evolves in a continuous Euclidean space and time, with a Markovian memory. The research workers, in statistical mechanics, in order to take into account the evolution of the process in a fractal space and discrete time with a non-Markovian memory, tend to searching for a different distribution function of classic and quantum physics. In this quest, for a more realistic description of the stochastic systems, for example, instead of the exponential function $e(z)$;

$$e_q(z) = [1 + (q-1)z]^{q-1} \tag{6}$$