Applied Mathematics and Computation 265 (2015) 275–291

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Periodic solutions in an epidemic model with diffusion and delay

Pan-Ping Liu*

Department of Mathematics, North University of China, Taiyuan, Shanxi 030051, People's Republic of China

ARTICLE INFO

PACS:
87.23.Cc
89.75.Kd
89.75.Fb
47.54.-r

Keywords:
Hopf bifurcation
Epidemic models
Time delay
Spatial diffusion

ABSTRACT

A spatial diffusion SI model with delay and Neumann boundary conditions are investigated. We derive the conditions of the existence of Hopf bifurcation in one dimension space. Moreover, we analyze the properties of bifurcating period solutions by using the normal form theory and the center manifold theorem of partial functional differential (PFDs) equations. By numerical simulations, we found that spatiotemporal periodic solutions can occur in the epidemic model with spatial diffusion, which verifies our theoretical results. The obtained results show that interaction of delay and diffusion may induce outbreak of infectious diseases.

1. Introduction

From the 2nd century A.D., the Roman Empire occurred Antonine plague [1], then the last few years there were outbreaks of SARS, H1N1 avian influenza, H7N9 avian influenza in the global world. These new emerging infectious diseases continuously increase every year, and the traditional infectious diseases recurrently outbreak [2–4]. Based on pathology of the disease, the route of transmission, and characteristics of the outbreak, we can establish the appropriate mathematical models and in turn, some reasonable suggestions can be provided for the prevention and effective control of infectious diseases.

With the study on mechanism of infectious diseases, many researchers found that the corresponding symptoms did not appear immediately after an individual was infected, and the individual took a period of time (namely exposed period) to show symptoms [5–8], such as rabies, AIDS, cholera and so on. In this case, time delay is needed to describe such phenomenon, which reflects that the changes of t moment not only depend on the state at t moment, but also are influenced by some factors before t moment [9–11].

Since an individual does not always stay in one place, conversely, it diffuses around. Therefore, the original reaction model becomes reaction–diffusion model. Moreover, the reaction–diffusion epidemic model with delay is able to better reflect the reality of disease [12–19]. For this kind of model, Thieme and Zhao analyzed a reaction–diffusion system with delay which described the spatial spread of a series bacterium and virus epidemic, then they obtained the traveling wave solutions and the existence of asymptotic propagation speed [20–22]. A spatial diffusion SEI epidemic model with delay was investigated by Kim and Lin, they obtained the sufficient conditions of the local and global asymptotical stability by using upper and lower solutions and its associated monotone iterations [23]. Li et al. studied pattern formation on an epidemic model with time delay, and concluded that the delay and diffusion can induce Turing pattern [10]. However, the studies of bifurcation behaviors on epidemic models with time delay and diffusion are still limited. Hence, in this paper, we studied the existence and the properties of Hopf bifurcation.

* Tel.: +8613513656782.
E-mail address: liuppzbdx@sina.com

http://dx.doi.org/10.1016/j.amc.2015.05.028
0096-3003/

The structure of this paper is as follows. In Section 2, we derive the characteristic equation of spatial diffusion SI model with delay, then the existence of the Hopf bifurcation is analyzed. In Section 3, by utilizing the normal form theory and the center manifold theorem, we get some properties of Hopf bifurcation. In Section 4, the results of the numerical simulation indicate that time delay can induce periodic outbreak of disease in spatial diffusion SI model with delay. Finally, we give some conclusions and discussions.

2. Mathematical modeling and the existence of Hopf bifurcation

2.1. Model formulation

A simple SI epidemic system produced surprising dynamics [24]. However, the authors did not consider the SI system includes both spatial factor and latent case. Thus, we will study the Hopf bifurcation on the basis of this system. The simple SI system is

$$\begin{aligned} \frac{dS}{dt} &= rN\left(1-\frac{N}{K}\right)-\beta\frac{SI}{N}-(\mu+m)S,\\ \frac{dI}{dt} &= \beta\frac{SI}{N}-(\mu+d)I. \end{aligned} \tag{1}$$

The total population (N) is divided into susceptible groups (S) and infectious groups (I), where r is the intrinsic growth rate, K is the carrying capacity of logistic equation, β denotes the contact transmission rate, μ is the natural mortality, d denotes the disease induced morality, m is the per-capital emigration rate of uninfectives.

The basic reproduction number is

$$R_0=\frac{\beta}{\mu+d},$$

and the basic demographic reproductive number is

$$R_d=\frac{r}{\mu+m}.$$

By re-scaling the above system, we have an epidemic system with spatial effect [25]:

$$\begin{aligned} \frac{\partial S}{\partial t} &= \upsilon R_d(S+I)(1-(S+I))-R_0\frac{SI}{S+I}-\upsilon S+d_1\nabla^2 S,\\ \frac{\partial I}{\partial t} &= R_0\frac{SI}{S+I}-I+d_2\nabla^2 I, \end{aligned} \tag{2}$$

where $\upsilon=\frac{\mu+m}{\mu+d}$ is the ratio of the average life-span of susceptibles to that of infections, d_1, d_2 are diffusion coefficients. In this paper, we consider one-dimension space domain, then the Laplacian operator $\nabla^2=\frac{\partial^2}{\partial x^2}$.

For most diseases, they have latent periods [26–28]. For example, Rabies may take several days or months to reach the infectious stage [29–32]. For such diseases, we need to introduce the time delay into the infected population. Moreover, we assume that the population can-not pass the boundary of the domain, and the outside population cannot enter this domain. As a result, we have the following system with Neumann boundary conditions:

$$\begin{cases} \frac{\partial S}{\partial t}=\upsilon R_d[S+I(t-\tau)][1-(S+I(t-\tau))]-R_0\frac{SI(t-\tau)}{S+I(t-\tau)}-\upsilon S+d_1\nabla^2 S,\\ \frac{\partial I}{\partial t}=R_0\frac{SI(t-\tau)}{S+I(t-\tau)}-I(t-\tau)+d_2\nabla^2 I,\quad t\ge 0, x\in(0,l\pi),\quad \left.\frac{\partial S}{\partial x}\right|_{x=0,l\pi}=0,\quad \left.\frac{\partial I}{\partial x}\right|_{x=0,l\pi}=0,\quad t\ge 0,\\ S(x,t)=\phi_1(x,t)\ge 0, I(x,t)=\phi_2(x,t)\ge 0,\quad (x,t)\in[0,l\pi]\times[-\tau,0]. \end{cases} \tag{3}$$

Assuming $\phi=(\phi_1,\phi_2)^T\in\wp=C([-\tau,0],X)$, $\tau>0$ and X is defined as

$$X=\left\{(S,I)^T: S,I\in W^{2,2}(0,l\pi);\ \left.\frac{\partial S}{\partial x}\right|_{x=0,l\pi}=\left.\frac{\partial I}{\partial x}\right|_{x=0,l\pi}=0\right\}$$

with the inner product $\langle\cdot,\cdot\rangle$.

2.2. Existence of Hopf bifurcation

At first, in the absence of diffusion and delay, system (3) is corresponding to the following system:

$$\begin{aligned} \frac{\partial S}{\partial t} &= \upsilon R_d(S+I)(1-(S+I))-R_0\frac{SI}{S+I}-\upsilon S,\\ \frac{\partial I}{\partial t} &= R_0\frac{SI}{S+I}-I. \end{aligned} \tag{4}$$