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Global asymptotic stability of nonautonomous Cohen–Grossberg neural network models with infinite delays

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ABSTRACT

For a general Cohen–Grossberg neural network model with potentially unbounded time-varying coefficients and infinite distributed delays, we give sufficient conditions for its global asymptotic stability. The model studied is general enough to include, as subclass, the most of famous neural network models such as Cohen–Grossberg, Hopfield, and bidirectional associative memory. Contrary to usual in the literature, in the proofs we do not use Lyapunov functionals. As illustrated, the results are applied to several concrete models studied in the literature and a comparison of results shows that our results give new global stability criteria for several neural network models and improve some earlier publications.

1. Introduction

The Cohen–Grossberg neural network models, first proposed and studied by Cohen and Grossberg [6] in 1983, have been the subject of an active research due to their extensive applications in various engineering and scientific areas such as neural-biology, population biology, and computing technology. The neural network model in [6] can be described by the following system of ordinary differential equations

$$x_i'(t) = -\rho_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{n} c_{ij}f_j(x_j(t)) + I_i\right], \quad t \geq 0,\ i = 1, \dots, n, \tag{1.1}$$

which includes, as a special case, the Hopfield neural network model

$$x_i'(t) = -b_i(x_i(t)) + \sum_{j=1}^{n} c_{ij}f_j(x_j(t)) + I_i, \quad t \geq 0,\ i = 1, \dots, n, \tag{1.2}$$

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studied by Hopfield [12] in 1984. In 1988, Kosko [15] presented a kind of neural network models, known as bidirectional associative memory (BAM), described by

$$\begin{cases} x_i'(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(y_j(t)) + I_i \\ \\ y_i'(t) = -y_i(t) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t)) + J_i \end{cases} \quad t \geq 0,\ i = 1, \dots, n. \tag{1.3}$$

The following generalization of the Cohen–Grossberg model (1.1),

$$x_i'(t) = -\rho_i(x_i(t)) \left[b_i(x_i(t)) + \sum_{j=1}^{n} f_{ij}(x_j(t)) \right], \quad t \geq 0,\ i = 1, \dots, n, \tag{1.4}$$

introduced in [19], includes most of neural network models as special cases, namely (1.1), (1.2), and (1.3).

In order to be more realistic, differential equations describing neural networks should incorporate time delays to take into account the synaptic transmission time among neurons, or, in artificial neural networks, the communication time among amplifiers. In 1989, Marcus and Westervelt [18] introduced for the first time a discrete delay in the Hopfield model (1.2), and they observed that the delay can destabilize the system. In fact, the delays can affect the dynamic behavior of neural network models [1] and, for this reason, stability of delayed neural networks has been investigated extensively ([2,3,7,8,13,14,16,19,20,23,25], and the references therein). Another relevant fact to take into account is that the neuron charging time, the interconnection weights, and the external inputs often change as time proceeds. Thus, the neural network models with temporal structure of neural activities should be introduced and investigated (see [5,23]).

For neural network models with time-varying coefficients, many authors assume the existence of an equilibrium point and study its global exponential stability (see [17,21,26]). Other authors study the global exponential stability of the models but, in these cases, they assume that the coefficients are bounded functions (see [4,23,24], and the references therein). Studies about global asymptotic stability of nonautonomous neural network models are few and the authors always assume bounded time-varying coefficients (see [14,16,22,25]). This paper aims at the global asymptotic stability of a general nonautonomous Cohen–Grossberg neural network model without assuming the existence of an equilibrium point or bounded time-varying coefficients.

Moreover, the models studied here have infinite delays and, when we are dealing with functional differential equations with infinite delays, the choice of an admissible Banach phase space requires special attention in order to have well-posedness of the initial value problem and standard results on existence, uniqueness, and continuation of solutions (see [9–11]). We note that many papers, dealing with neural networks with unbounded delays, do not provide an explicit phase space, much less the problem of such a suitable choice, and it is known that the choice of the phase space is not the same when we are studying the global exponential stability or we are studying the global asymptotic stability [8].

After the introduction, the present paper is divided into three sections. Section 2 is a preliminary section, where some notation and definition are introduced and the phase space for the models is presented. In Section 3, we present the results on global asymptotic stability of a general nonautonomous Cohen–Grossberg neural network model with infinite delays, which includes most of neural network models. It is important to note that most studies use a type of Lyapunov functional to obtain results on global attractivity (see [4,5,14,16,24,25]). Instead, here we use the techniques described in the work of Oliveira [20], rather than a Lyapunov functional approach. In fact, we consider a Cohen–Grossberg model in general setting for which, assuming the existence of instantaneous negative feedbacks which dominate the delay effect, we prove that all solutions are defined on $\mathbb{R}$ and then we prove the global asymptotic stability of the model. Finally, in Section 4, we illustrate the results with well-known neural network examples and we compare our results with the literature, presenting new stability criteria.

2. Notations and definitions

We denote by $BC = BC((-\infty, 0]; \mathbb{R}^n)$ the space of bounded and continuous functions, $\phi : (-\infty, 0] \to \mathbb{R}^n$, equipped with the norm $||\phi|| = \sup_{s \leq 0} |\phi(s)|$, where $|\cdot|$ is the maximum norm in $\mathbb{R}^n$, i.e. $|x| = \max\{|x_i| : i = 1, \dots, n\}$ for $x = (x_1, \dots, x_n) \in \mathbb{R}^n$. For $a \in \mathbb{R}^n$, we also use a to denote the constant function $\varphi(s) = a$ in BC. A vector $c = (c_1, \dots, c_n) \in \mathbb{R}^n$ is said to be positive if $c_i > 0$ for all $i \in \{1, \dots, n\}$ and in this case we write $c > 0$.

For an open set $D \subseteq BC$ and $f : [0, +\infty) \times D \to \mathbb{R}^n$ a continuous function, consider the functional differential equation (FDE) given in general setting by

$$x'(t) = f(t, x_t), \quad t \geq 0, \tag{2.1}$$

where, as usual, x_t denotes the function $x_t : (-\infty, 0] \to \mathbb{R}^n$ defined by $x_t(s) = x(t+s)$ for $s \leq 0$.

It is well-known that the Banach space BC is not an admissible phase space for (2.1), in the sense of [9], thus the standard existence, uniqueness, continuous dependence type results are not available. Instead of BC, we consider the admissible