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A spline collocation method for Fredholm–Hammerstein integral equations of the second kind in two variables

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ABSTRACT

We consider Fredholm–Hammerstein integral equations of the second kind over a rectangular region in plane. As in Kumar and Sloan (1987) [5], we reformulate it into an equivalent integral equation. For the alternative equation, we triangulate the rectangular domain and on each triangle use a collocation method based on constant spline approximation. We discuss the convergence of the approximate solutions and conclude the paper with numerical examples.

1. Introduction

Integral equations have a wide variety of applications in many fields of engineering, physics, chemistry, biology, mechanics, astronomy. They provide an important tool for modeling various phenomena and processes occurring in heat conducting radiation, elasticity, electrostatics, radiative transfer, computer graphics, realistic illumination, particle transport problems of astrophysics, chemical kinetics, chemical reactor theory, economics, theory of communication systems, magnetohydrodynamics, quantum mechanics and many other areas (for more applications of integral equations, see [11]). Also, many initial and boundary value problems associated with ordinary and partial differential equations can be reformulated as integral equations (see [10]).

Numerical solutions of nonlinear Fredholm integral equations of the second kind have been studied extensively, through a variety of methods. Projection methods – collocation [2,5,9] and Galerkin [3] – and Nyström methods [7] are among the most popular ones. Also, recent results were obtained using kernel methods [4] and Adomian decomposition methods [6]. For more details on approximating methods, see [1].

In this paper, we consider a Fredholm–Hammerstein integral equation of the type

$$u(x, y) = \int_D k(x, y, s, t)g(s, t, u(s, t))ds\, dt + f(x, y), \ \ (x, y) \in D = [a, b] \times [c, d],$$

where $g : D \times \mathbb{R} \to \mathbb{R}$ is a continuous nonlinear function. Other smoothness assumptions will be made on k, g and f later on.

For simplicity, we use the shorter notation $q = (x, y)$ and $w = (s, t)$. So, we have the integral equation

$$u(q) = \int_D k(q, w)g(w, u(w))dw + f(q), \ \ q \in D = [a, b] \times [c, d]. \tag{1.1}$$

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Remark 1.1. The ideas we present here work for other closed regions $D \subseteq \mathbb{R}^2$, as well, but for simplicity, we restrict ourselves to the case $D = [a, b] \times [c, d]$.

As in [5], let

$$v(w) := g(w, u(w)).$$

Then v and u must satisfy the integral equations

$$v(q) = g\left(q, \int_D k(q, w)v(w)dw + f(q)\right), \; q \in D \tag{1.2}$$

and

$$u(q) = \int_D k(q, w)v(w)dw + f(q), \; q \in D, \tag{1.3}$$

respectively.

Under certain assumptions, an approximation scheme applied to Eq. (1.2) will lead to an approximate solution of (1.3). Throughout this paper, we assume the following conditions are satisfied:

C1 The integral operator K: $C(D) \to C(D)$ defined by

$$(Ku)(q) = \int_D k(q, w)g(w, u(w))dw$$

is completely continuous;

C2 The equation $u = Ku + f$ has an isolated solution u^* with non-zero index, which is assumed to be smooth enough;

C3 The function $f \in C(D)$;

C4 The derivative $g_u(w, u)$ exists and is continuous on $D \times \mathbb{R}$.

We briefly describe the general collocation method framework. Suppose $\{\tau_1, \ldots, \tau_m\} \subset D$ are nodes in D and $\{l_{m1}, \ldots, l_{mm}\}$ is a set of functions defined on D such that $l_j(\tau_i) = \delta_{ij}$, $1 \le i, j \le m$. Let $P_m : D \to D_m = \text{span}\{l_{m1}, \ldots, l_{mm}\}$ be the interpolatory projection operator defined by

$$(P_m u)(q) = \sum_{j=1}^{m} u(\tau_j) l_{mj}(q), \; q \in D. \tag{1.4}$$

Then P_m is a bounded linear operator with norm

$$||P_m|| = \sup_{q \in D} \sum_{j=1}^{m} |l_{mj}(q)|.$$

Assume that

$$\lim_{m \to \infty} ||u - P_m u|| = 0, \text{ for all } u \in C(D). \tag{1.5}$$

Recall that u^* is the isolated solution of (1.1) that we are trying to approximate and let v^* be the corresponding solution of (1.2). Using P_m, we define an approximation of v^* by

$$v_m(q) = P_m v(q) = \sum_{j=1}^{m} v_m(\tau_j) l_{mj}(q), \tag{1.6}$$

where the values $\{v_m(\tau_j)\}_{j=1}^m$ are determined by forcing Eq. (1.2) to be true at the collocation points, i.e. from the system

$$v_m(\tau_i) = g\left(\tau_i, \sum_{j=1}^{m} v_m(\tau_j) \int_D k(\tau_i, w) l_{mj}(w)dw + f(\tau_i)\right), i = 1, \ldots, m. \tag{1.7}$$

This leads to the approximate solution of (1.3)

$$u_m(q) = \int_D k(q, w)v_m(w)dw + f(q) = \sum_{j=1}^{m} v_m(\tau_j) \int_D k(q, w) l_{mj}(w)dw + f(q). \tag{1.8}$$

Regarding these, we have the following result:

Theorem 1.2. ([5, Theorem 2]) *Assume conditions* C1-C4 *hold and that the operator P_m defined in (1.4) satisfies (1.5). Then*

$$||v_m - v^*|| \to 0, \; ||u_m - u^*|| \to 0, \text{ as } m \to 0.$$