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Long-time behavior of a suspension bridge equations with past history

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ABSTRACT

In this paper, we study a suspension bridge equation with memory effects. For the suspension bridge equation without memory, there are many classical results. Existing results mainly devoted to existence and uniqueness of a weak solution, energy decay of solution and existence of global attractors. However the existence of global attractors for the suspension bridge equation with memory was no yet considered. The object of the present paper is to provide some results on the well-posedness and long-time behavior to the suspension bridge equation when the unique damping mechanism is given by the memory term.

1. Introduction

In this paper, we study the global attractor for the suspension bridge equation with memory

$$u_{tt}+\alpha\Delta^2u-\int_0^\infty \mu(s)\Delta^2u(t-s)ds+ku^+ +f(u)=g(x) \text{ in } \Omega\times\mathbb{R}^+, \tag{1.1}$$

$$u=0,\ \ \Delta u=0 \text{ on } \Gamma\times\mathbb{R}, \tag{1.2}$$

$$u(x,\tau)=u_0(x,\tau),\ u_t(x,\tau)=\partial_t u_0(x,\tau),\ \ (x,\tau)\in\Omega\times(-\infty,0], \tag{1.3}$$

where Ω is a bounded domain of $\mathbb{R}^2$ with a smooth boundary Γ. α is positive constant, k is a spring constant, μ is the memory kernel, and f, g are forcing terms. The force $u^+=\max\{u,0\}$ is the positive part of u and $u_0:\Omega\times(-\infty,0]\to\mathbb{R}$ is the prescribed past history of u.

From the physics point of view, the suspension bridge equation describes the transverse deflection of the roadbed in the vertical plane. The suspension bridge equations were presented by Lazer and McKenna as new problems in the field of nonlinear analysis [12]. The global existence and the global attractors of solutions for the suspension bridge equation were studied in [1,2,14–16,18,20,22] and references therein. Ma and Zhong [15] investigated the existence of global attractors of the weak solutions for the suspension bridge equations. Zhong et al. [22] proved the existence of strong solutions and global attractors for the suspension bridge equations. Park and Kang [18,20] obtained the existence of pullback attractor for the non-autonomous suspension bridge equations and the existence of global attractors for the suspension bridge equations with nonlinear damping, respectively. For the coupled suspension bridge equations, Ahmed and Harbi [1] studied the existence of weak solution.

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Ma and Zhong [14,16] showed the existence of strong solutions and global attractors for the coupled suspension bridge equation. Recently, Kang [11] investigated the pullback $\mathcal{D}$-attractors for the non-autonomous coupled suspension bridge equations when external terms are unbounded in a phase space. However the existence of global attractors for the suspension bridge equation with memory was not yet considered. The object of the present paper is to provide some results on the well-posedness and long-time behavior to the suspension bridge equation when the unique damping mechanism is given by the memory term.

Fabrizio et al. [6] discussed a novel approach to the mathematical analysis of equations with memory. Giorgi et al. [7] proved the global attractor of semi-linear hyperbolic equation with linear memory, arising in the theory of isothermal viscoelasticity

$$\begin{cases} u_{tt} - k(0)\Delta u - \int_0^\infty k'(0)\Delta u(t-s)ds + g(u) = f, & \text{in } \Omega \times \mathbb{R}^+, \\ u(x,t) = 0, & \text{on } \partial\Omega \times \mathbb{R}, \\ u(x,t) = u_0(x,t), & (x,t) \in \Omega \times (-\infty, 0]. \end{cases}$$

Ma and Zhong [13] obtained the global attractor of the hyperbolic equations with linear memory and linear damping. Later, Park and Kang [19] generalized the results of [7]. By adapting a classical argument by Dafermos [5], they studied the global attractor of semi-linear hyperbolic equation with linear memory and nonlinear damping. Silva and Ma [9,10] proved the exponential stability and global attractor of plate equations with memory and perturbation of p-Laplacian type. Recently, Araújo et al. [17] considered the long-time behavior of a quasilinear viscoelastic equation with past history.

Motivated by these results, we study the existence of global attractors for the suspension bridge equation with past history.

We end this section by introducing the relative displacement memory that will transform Eq. (1.1) into an autonomous system. As in [5,7], we add a new variable η to the system corresponding to the relative displacement memory, namely

$$\eta = \eta^t(x,s) = u(x,t) - u(x,t-s),\ (x,s) \in \Omega \times \mathbb{R}^+,\ t \geq 0. \tag{1.4}$$

By formal differentiation in (1.4) we obtain

$$\eta_t^t(x,s) = -\eta_s^t(x,s) + u_t(x,t),\ (x,s) \in \Omega \times \mathbb{R}^+,\ t \geq 0.$$

and we take as initial condition $(t = 0)$

$$\eta^0(x,s) = u_0(x,0) - u_0(x,-s),\ (x,s) \in \Omega \times \mathbb{R}^+.$$

By assuming that $\mu \in L^1(\mathbb{R}^+)$ and taking $\alpha = 1 + \int_0^\infty \mu(s)ds$, the original problem (1.1)–(1.3) can be transformed into the equivalent system

$$u_{tt} + \Delta^2 u + \int_0^\infty \mu(s)\Delta^2\eta^t(s)ds + ku^+ + f(u) = h(x) \text{ in } \Omega \times \mathbb{R}^+, \tag{1.5}$$

$$\eta_t = -\eta_s + u_t \text{ in } \Omega \times \mathbb{R}^+ \times \mathbb{R}^+, \tag{1.6}$$

with boundary conditions

$$u = \Delta u = 0 \text{ on } \Gamma \times \mathbb{R}^+,\ \eta = \Delta\eta = 0 \text{ on } \Gamma \times \mathbb{R}^+ \times \mathbb{R}^+, \tag{1.7}$$

and initial conditions

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ \eta^t(x,0) = 0,\ \eta^0(x,s) = \eta_0(x,s), \tag{1.8}$$

where

$$\begin{cases} u_0(x) = u_0(x,0), & x \in \Omega, \\ u_1(x) = \partial_t u_0(x,t)|_{t=0}, & x \in \Omega, \\ \eta_0(x,s) = u_0(x,0) - u_0(x,-s), & (x,s) \in \Omega \times \mathbb{R}^+. \end{cases}$$

This paper is organized as follows: In Section 2, we give some preparations for our consideration. We also present an overview of the abstract results in the theory of infinite dimensional dynamical systems that will be used. In Section 3, we prove the existence of a global attractor.

2. Preliminaries

We begin this section introducing notations and some hypotheses. Throughout this paper we use standard functional space and denote that $(\cdot,\cdot)$ is $L^2(\Omega)$-inner product and $||\cdot||_p$ is $L^p(\Omega)$ norm. Let

$$H = V_0 = L^2(\Omega),\ V = V_1 = H^2(\Omega) \cap H_0^1(\Omega),$$

equipped with respective inner product and norm,

$$(u,v)_V = (\Delta u, \Delta v) \quad \text{and} \quad ||u||_V = ||\Delta u||_2.$$

In order to consider the relative displacement η as a new variable, one introduces the weighted L^2-space

$$L_\mu^2(\mathbb{R}^+;V) = \{\eta : \mathbb{R}^+ \to V \mid \int_0^\infty \mu(s)||\eta(s)||_V^2 ds < \infty\},$$