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Pullback attractor for differential evolution inclusions with infinite delays

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ABSTRACT

We analyze the existence of pullback attractor for non-autonomous differential inclusions with infinite delays in Banach spaces by using measures of noncompactness. The obtained results can be applied to control systems driven by semilinear partial differential equations and multivalued feedbacks.

1. Introduction

Let $(X, \|\cdot\|)$ be a Banach space. We consider the following problem:

$$u'(t) \in Au(t) + F(t, u_t),\ t \geq \tau, \tag{1.1}$$

$$u_\tau(s) = \varphi^\tau(s),\ s \in (-\infty, 0], \tag{1.2}$$

where the state function u takes values in X, A is a closed linear operator generating a C_0-semigroup $\{S(t)\}_{t\geq 0}$ on X, F is a multi-valued function defined on $\mathbb{R} \times \mathcal{B}$, u_t is the history of the state function up to the time t, *i.e.* $u_t(s) = u(t+s)$ for $s \in (-\infty, 0]$. The function $\varphi^\tau \in \mathcal{B}$ takes values in X and plays the role of initial datum. Here $\mathcal{B}$ is a phase space that will be described in the next section.

It is known that differential inclusions (DIs) like (1.1) come from various problems. As the first one, we recall the study of Filippov [13], where differential equations with discontinuous nonlinearity are *regularized* to DIs. Differential inclusions are also arisen from control systems in which control factor is taken in the form of feedbacks. Let us mention, as a remarkable aspect, that some classes of differential variational inequalities can be converted to differential inclusions (see [22]). For a comprehensive study of DIs, we refer to the works presented in [4,12,16].

One of the most important and interesting problems related to DIs is to study the stability of solutions. Since, the question of uniqueness of solutions to DIs is no longer addressed, the Lyapunov theory for stability is not a suitable choice. Thanks to the theories of attractors for multivalued semiflows/processes given in [7,8,10,20,21], one can find a global attractor for semi-flows/processes governed by solutions of DIs, which is a compact set attracting all solutions as the time goes to infinity in some

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contexts. Some deployments of these theories for differential inclusions and differential equations without uniqueness can be found in [2,3,8–10,18,24,25].

In the present paper, we first prove the global solvability for (1.1)–(1.2) under a general setting. Then by giving a new criterion ensuring the asymptotic compactness for multivalued non-autonomous dynamical systems (MNDS), we show that the MNDS generated by (1.1)–(1.2) admits a pullback attractor in case the phase space $\mathcal{B} = \mathcal{C}_\gamma$ (see Section 2 for the description). Our motivation comes from the fact that the study for MNDS governed by DIs with unbounded delays is little-known. It should be noted that the appearance of infinite delays causes some technical difficulties in proving the dissipativeness as well as the asymptotic compactness of MNDS. These difficulties are due to the complication of phase spaces. We refer the reader to some recent results established in [6,8] for the case when F is single-valued. The main aim of our work is to deal with the case of multivalued nonlinearities by using a concise argument based on measures of noncompactness.

The rest of our work is organized as follows. In the next section, we recall some notions and facts related to measures of noncompactness and MNDS. Section 3 is devoted to proving the solvability on $(-\infty, \tau + T], T > 0$. In Section 4, with some additional assumptions, we prove the existence of a compact invariant pullback attractor in $\mathcal{C}_\gamma$ for the MNDS governed by our system. The last section presents an application of the abstract results to a feedback control problem associated with partial differential equations.

2. Preliminaries

2.1. Phase space

Let $(\mathcal{B}, |\cdot|_{\mathcal{B}})$ be a semi-normed linear space of functions mapping from $(-\infty, 0]$ into a Banach space X. The definition of the phase space $\mathcal{B}$, introduced in [15], is based on the following axioms. If $v : (-\infty, \sigma + T] \to X$, where $\sigma \in \mathbb{R}$ and T is a positive number, is a function such that $v|_{[\sigma,\sigma+T]} \in C([\sigma, \sigma + T]; X)$ and $v_\sigma \in \mathcal{B}$, then

(B1) $v_t \in \mathcal{B}$ for $t \in [\sigma, \sigma + T]$;
(B2) the function $t \mapsto v_t$ is continuous on $[\sigma, \sigma + T]$;
(B3) $|v_t|_{\mathcal{B}} \le K(t - \sigma) \sup\{\|v(s)\| : \sigma \le s \le t\} + M(t - \sigma)|v_\sigma|_{\mathcal{B}}$ for each $t \ge \sigma$, where K, M: $[0, \infty) \to [0, \infty)$, K is continuous, M is locally bounded and they are independent of v.

In this work, we need an additional assumption on $\mathcal{B}$:

(B4) there exists $\ell > 0$ such that $\|\phi(0)\|_X \le \ell|\phi|_{\mathcal{B}}$, for all $\phi \in \mathcal{B}$.

A typical example of phase spaces is C_γ defined as follows:

$$C_\gamma = \{\phi \in C((-\infty, 0]; X) \text{ such that } \lim_{\theta \to -\infty} e^{\gamma\theta}\phi(\theta) \text{ exists in } X\}.$$

If $\gamma > 0$ then C_γ is a Banach space with norm

$$|\phi|_\gamma = \sup_{\theta \le 0} e^{\gamma\theta}\|\phi(\theta)\|.$$

In this case $K(t) = 1, M(t) = e^{-\gamma t}$, for all $t \ge 0$. For more examples of phase space, see [14].

2.2. Condensing multivalued maps

Let E be a Banach space. We denote by 2^E the collection of all subsets of E and use the following notations:

$$\mathcal{P}(E) = \{A \in 2^E : A \ne \emptyset\},$$
$$\mathcal{P}_b(E) = \{A \in \mathcal{P}(E) : A \text{ is bounded}\},$$
$$\mathcal{P}_c(E) = \{A \in \mathcal{P}(E) : A \text{ is closed}\},$$
$$Kv(E) = \{A \in \mathcal{P}(E) : A \text{ is compact and convex}\}.$$

We recall the Hausdorff measure of noncompactness (MNC) $\chi(\cdot)$, which is defined as follows

$$\chi(B) = \inf\{\epsilon > 0 : B \text{ has a finite } \epsilon\text{-net}\}.$$

For $\mathcal{T} \in \mathcal{L}(E)$, the space of bounded linear operators on E, we define the χ-norm of $\mathcal{T}$ as follows (see, e.g. [1])

$$\|\mathcal{T}\|_\chi = \inf\{\beta > 0 : \chi(\mathcal{T}(B)) \le \beta\chi(B) \text{ for all } B \in \mathcal{P}_b(E)\}.$$

It is noted that the χ-norm of $\mathcal{T}$ can be formulated by

$$\|\mathcal{T}\|_\chi = \chi(\mathcal{T}(\mathbf{B}_1)) = \chi(\mathcal{T}(\mathbf{S}_1)),$$

where $\mathbf{B}_1$ and $\mathbf{S}_1$ are a unit ball and a unit sphere in E, respectively. It is known that

$$\|\mathcal{T}\|_\chi \le \|\mathcal{T}\|,$$

where the last norm is understood as the operator norm in $\mathcal{L}(E)$. Obviously, $\mathcal{T}$ is a compact operator iff $\|\mathcal{T}\|_\chi = 0$.