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A Robust and accurate Riemann solver for a compressible two-phase flow model

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ABSTRACT

In this paper we analyze the Riemann problem for the widely used drift-flux two-phase flow model. This analysis introduces the complete information that is attained in the representation of solutions to the Riemann problem. It turns out that the Riemann waves have rarefactions, a contact discontinuity and shocks. Within this respect, an exact Riemann solver is developed to accurately resolve and represent the complete wave structure of the gas-liquid two-phase flows. To verify the solver, a series of test problems selected from the literature are presented including validation against independent numerical simulations where the solution of the Riemann problem is fully numerical. In this framework the governing equations are discretized by finite volume techniques facilitating the application Godunov methods of centred-type. It is shown that both analytical and numerical results demonstrate the broad applicability and robustness of the new exact Riemann solver.

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1. Introduction

The subject of multiphase flows has become increasingly important in engineering design and operations such as wellbores, chemical reactors and nuclear industry. Mathematical modelling of two-phase flows within such context depend on the theoretical details of the problem in hand and on the range of approaches and computational resources available for such flows. However, many studies towards two-phase flows have been presented in the literature. These in general include empirical correlations, homogeneous models and mechanistic models [1–3]. In practical situations, mechanistic models are often considered as they are known to provide more details about the physics of each fluid pattern. The mathematical description of such models, however, is not significantly different from those less demanding two-phase flow models. The challenge among these models is always related to the hyperbolicity and conservativity nature of the governing equations [4–6]. Despite the challenges, interest in these models have remained high and the number of research developments has grown steadily. See for example [7–10] and references therein.

One of the simplest mechanistic models is the drift-flux model. The drift-flux model, or approach, results from averaging of the mathematical equations for single-phase flow and consists of a continuity equation for each phase, and a momentum equation for the mixture. The drift-flux model was first proposed by Zuber and Findlay [1] and has been advanced by many researchers for two-phase flow analysis (see, *e.g.* [10–13] and the references therein). This is due to its simplicity, transparency and accuracy in various industrial applications such as thermal-hydraulic analysis codes, wellbores and the reservoir simulators

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[9,14–18]. Furthermore, the number of publications related to such a model has increased recently and some fundamental and challenging issues still remained unsolved, such as the Riemann problem to the model equations. The drift-flux model takes into account the slip motion between phases by a constitutive relation [1,14]. This relation makes it very challenging and difficult to aim for a closed form, analytical, solutions. Due to the complicated nature of the resulting differential equations, early analytical solutions were based on certain simplifications such as the steady-state homogeneous flow (see for example [14,18–22]). Our objective in this paper, therefore, is to move forward beyond such analysis and explore general properties of the drift-flux model. Furthermore, we present an analytical procedure to solve the drift-flux model of unsteady-state and compressible two-phase flows. The solution is based on the Riemann problem for the model equations. As regard to the Riemann problem, it has achieved a considerable importance in two-phase fluid flow problems and several authors have proposed numerical methods based on solving the Riemann problem solutions. In [23] an exact solver for homogeneous Baer–Nunziato model (see, [24]) was developed on the basis of inverse solution to the Riemann problem. That is, the authors fixed the solution of the Riemann problem and then they look for initial data which are compatible with the assumed wave configuration. The authors in [25] have also developed a solution to the Riemann problem for the Baer–Nunziato model of compressible two-phase flows for given initial data. Another exact solution was also developed in [26] for the Riemann problem for the five equation two-phase non-conservative model of [7]. In [27] a direct, non-linear Riemann solver for two-phase flow in gas-liquid mixture was developed and employed locally within the framework of finite volume upwind Godunov methods.

In this paper we investigate the Riemann problem for a hyperbolic system of partial differential equations governing the known drift-flux model for two-phase flows (see, [28] and the references therein). Unlike single equation methods noted in the conventional ideal gas-dynamics, the present approach is based on the system of two non-linear equations imposing the equality of common velocity and the pressure on the two sides of contact discontinuity. We solve the non-linear algebraic system using Newton–Raphson iterative procedure with a stopping criteria where the relative error is less than 10^{-8}; the initial guess for the intermediate unknown physical quantity is taken to be the average of left and right state. The solution to the Riemann problem provides a building block for a wide class of numerical methods such as Godunov methods [29]. Finally, the exact solution to the Riemann problem is an invaluable test case which is useful in assessing the performance of numerical methods as we shall see in the current paper.

The structure of this paper as follows. In Section 2, we recall the drift-flux model. In Section 3, we discuss the properties of elementary waves, namely, rarefaction waves, shock waves and contact discontinuities of the Riemann problem associated with the model equations. In Section 4 , we give the full details for the construction of Riemann solutions. In Section 5, we describe the numerical method used to validate the developed Riemann solver. A number of test cases conducted during our investigation are presented in Section 6. These test cases are based on the exact solution to the Riemann problem. Finally, conclusions are given in Section 7.

2. Governing equations

The two-phase fluid flow model considered in this paper is based on the Eulerian description of both phases. Such a description has been widely used in the literature where balance equations represent the evolution of mass, momentum and energy for each phase. For isentropic one-dimensional flows these equations are given by two mass and two momentum balance equations for each phase as follows:

$$\frac{\partial}{\partial t}(\rho_g\alpha_g)+\frac{\partial}{\partial x}(\rho_g\alpha_g u_g)=S_1, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_l\alpha_l)+\frac{\partial}{\partial x}(\rho_l\alpha_l u_l)=S_2, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho_g\alpha_g u_g)+\frac{\partial}{\partial x}\left(\rho_g\alpha_g u_g^2\right)+\frac{\partial}{\partial x}(\alpha_g p_g)=S_3, \tag{3}$$

$$\frac{\partial}{\partial t}(\rho_l\alpha_l u_l)+\frac{\partial}{\partial x}\left(\rho_l\alpha_l u_l^2\right)+\frac{\partial}{\partial x}(\alpha_l p_l)=S_4. \tag{4}$$

Index g in the above system is referred to the gas and l to the liquid phase; x is the space coordinate and t is the time; ρ_j, u_j, p_j and α_j are the density, velocity, pressure and volume fraction of phase j $(j=g,l)$, respectively. The volume fractions are subject to the constraint

$$\alpha_g+\alpha_l=1. \tag{5}$$

The source terms on the right hand side of Eqs. (1)–(4) depend on the modeled two-phase flow pattern. They consists of constitutive relations for the interface transfers of interest. For the purposes of this paper, we have neglected these source terms and thus it is unnecessary to carry them along in the current theoretical investigation. The above system is closed with the equation of state (EOS) for each phase. In particular, isentropic laws are considered in the following form

$$p_g=p_g(\rho_g)\quad\text{and}\quad p_l=p_l(\rho_l). \tag{6}$$

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