Applied Mathematics and Computation 265 (2015) 759–767

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Applying the modified block-pulse functions to solve the three-dimensional Volterra–Fredholm integral equations

Farshid Mirzaee*, Elham Hadadiyan

Faculty of Mathematical Sciences and Statistics, Malayer University, P.O. Box 65719-95863, Malayer, Iran

ARTICLE INFO

MSC:
45B05
45D05
65D30
65R20

Keywords:
Three-dimensional
Volterra–Fredholm
Modification
Block-pulse functions
Error analysis

ABSTRACT

The main aim of this work is to give further studies for the multi-dimensional integral equations. In this work, we solve special types of the three-dimensional Volterra–Fredholm linear integral equations of the second kind via the modified block-pulse functions. Some theorems are included to show convergence and advantage of the method. We solve some examples to investigate the applicability and simplicity of the method. The numerical results confirm that the method is efficient and simple.

1. Introduction

The multi-dimensional integral equation is an integral equation in which the integration is carried out with respect to multiple variables. In this case, the unknown function depends on more than one independent variable. Moreover, the multi-dimensional integral equations can be found in many technologies, mechanics and biology. Many problems of fracture mechanics, aerodynamics, the theory of porous filtering, antenna problems in electromagnetic theory and others can be formulated as multi-dimensional integral equations of the first, second and third kind [1].

Solution methods for the multi-dimensional integral equations are very significant since they appear in the mathematical formulation. Because these equations are usually difficult to solve analytically, the aim in the present research is to develop an accurate as well as easy to implement analytic solution scheme to treat such equations. Numerous works have been focusing on the development of more advanced and efficient methods for solving Volterra–Fredholm integral equations. The literature on the numerical solution methods of such equations is fairly extensive [2–9]. But the analysis of computational methods for two-dimensional integral equations seem to have been discussed in only a few papers [10–19].

Block-pulse functions have been studied and applied extensively as a basic set of functions for signal and function approximations. All these studies and applications show that block-pulse functions have definite advantages for solving problems involving integrals and derivatives due to their clearness in expressions and their simplicity in formulations.

This paper is divided into the following sections. The properties of modified three-dimensional block-pulse functions (M3D-BFs) are presented in the next section. Section 3 briefly reviews the description of the method based on M3D-BFs for solving the special types of the multi-dimensional Volterra–Fredholm integral equations. In Section 4, theorems are provided for convergence analysis. Numerical results are given in Section 5. Section 6 consists of a brief conclusion.

* Corresponding author. Tel.: +98 81 32355466; fax: +98 81 32355466.
E-mail address: f.mirzaee@malayeru.ac.ir, f.mirzaee@iust.ac.ir (F. Mirzaee).

http://dx.doi.org/10.1016/j.amc.2015.05.125
0096-3003/

2. Preliminaries

2.1. About M3D-BFs bases

Now we describe the fundamental idea of M3D-BFs. We define the $(m+1)^3$-set of M3D-BFs over district $D = [0,1) \times [0,1) \times [0,1)$ as follows

$$\phi_{i_1,i_2,i_3}(x,y,z) = \begin{cases} 1 & (x,y,z) \in D_{i_1,i_2,i_3}, \\ 0 & \text{otherwise}, \end{cases} \quad i_1,i_2,i_3 = 0(1)m,$$

where

$$D_{i_1,i_2,i_3} = \left\{(x,y,z) | x \in I_{i_1,\varepsilon}, y \in I_{i_2,\varepsilon}, z \in I_{i_3,\varepsilon}\right\},$$

$$I_{\alpha,\varepsilon} = \begin{cases} [0, h-\varepsilon) & \alpha = 0, \\ [\alpha h - \varepsilon, (\alpha+1)h - \varepsilon) & \alpha = 1(1)(m-1), \\ [1-\varepsilon, 1) & \alpha = m, \end{cases}$$

and $h = \frac{1}{m}$ which m is an arbitrary positive integer. Since each M3D-BF takes only one value in its subregion, the M3D-BFs can be expressed by the three modified one-dimensional block-pulse functions (M1D-BFs)

$$\phi_{i_1,i_2,i_3}(x,y,z) = \phi_{i_1}(x)\phi_{i_2}(y)\phi_{i_3}(z),$$

where $\phi_{i_1}(x)$, $\phi_{i_2}(y)$ and $\phi_{i_3}(z)$ are the M1D-BFs related to the variables x, y and z [20]. The M3D-BFs in the subregion $(x,y,z) \in D$ are disjointed as

$$\phi_{i_1,i_2,i_3}(x,y,z)\phi_{j_1,j_2,j_3}(x,y,z) = \delta_{i_1,i_2,i_3}^{j_1,j_2,j_3}\phi_{i_1,i_2,i_3}(x,y,z),$$

and are orthogonal as

$$\int_0^1\int_0^1\int_0^1 \phi_{i_1,i_2,i_3}(x,y,z)\phi_{j_1,j_2,j_3}(x,y,z)dzdydx = \delta_{i_1,i_2,i_3}^{j_1,j_2,j_3}\triangle(I_{i_1,\varepsilon})\triangle(I_{i_2,\varepsilon})\triangle(I_{i_3,\varepsilon}),$$

where $i_1, i_2, i_3, j_1, j_2, j_3 = 0(1)m$, $\delta_{i_1,i_2,i_3}^{j_1,j_2,j_3}$ is Kronecker delta and $\triangle(I_{i_1,\varepsilon})$, $\triangle(I_{i_2,\varepsilon})$ and $\triangle(I_{i_3,\varepsilon})$ are length of intervals $I_{i_1,\varepsilon}$, $I_{i_2,\varepsilon}$ and $I_{i_3,\varepsilon}$, respectively.

Now, consider the first $(m+1)^3$ terms of M3D-BFs and write them concisely as $(m+1)^3$-vector

$$\Phi_{m,\varepsilon}(x,y,z) = [\phi_{0,0,0}(x,y,z), \ldots, \phi_{0,0,m}(x,y,z), \ldots, \phi_{0,m,m}(x,y,z), \ldots, \phi_{m,m,m}(x,y,z)]^T, \tag{1}$$

where $(x,y,z) \in D$. The above representation and disjointness property follows

$$\Phi_{m,\varepsilon}(x,y,z)\Phi_{m,\varepsilon}^T(x,y,z) = diag(\Phi_{m,\varepsilon}(x,y,z)). \tag{2}$$

It is clear that

$$\int_0^x\int_0^y\int_0^z \Phi_{m,\varepsilon}(s,t,r)drdtds = \left(\int_0^x \Phi_{m,\varepsilon}(s)ds\right) \otimes \left(\int_0^y \Phi_{m,\varepsilon}(t)dt\right) \otimes \left(\int_0^z \Phi_{m,\varepsilon}(r)dr\right)$$
$$\simeq (p_1\Phi_{m,\varepsilon}(x)) \otimes (p_1\Phi_{m,\varepsilon}(y)) \otimes (p_1\Phi_{m,\varepsilon}(z)) = (p_1 \otimes p_1 \otimes p_1)\Phi_{m,\varepsilon}(x,y,z) = P1\Phi_{m,\varepsilon}(x,y,z), \tag{3}$$

and

$$\int_0^1\int_0^1\int_0^1 \Phi_{m,\varepsilon}(s,t,r)\Phi_{m,\varepsilon}^T(s,t,r)drdtds = \left(\int_0^1 \Phi_{m,\varepsilon}(s)\Phi_{m,\varepsilon}^T(s)ds\right) \otimes \left(\int_0^1 \Phi_{m,\varepsilon}(t)\Phi_{m,\varepsilon}^T(t)dt\right)$$
$$\otimes\left(\int_0^1 \Phi_{m,\varepsilon}(r)\Phi_{m,\varepsilon}^T(r)dr\right) = p_2 \otimes p_2 \otimes p_2 = P2, \tag{4}$$

where

$$\Phi_{m,\varepsilon}(x) = [\phi_0(x), \phi_1(x), \ldots, \phi_m(x)]^T,$$