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Four point interpolatory-corner cutting subdivision

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A B S T R A C T

This paper presents a new curve algorithm called four point interpolatory-corner cutting subdivision for generating curves that interpolate some given vertices and approximate the other vertices. By four-point interpolatory-corner cutting subdivision, only the vertices specified to be interpolated are fixed and the other vertices are updated at each refinement step. Compared to the four-point interpolation subdivision scheme, it is expected to have improved behavior, and compared to the cutting smooth subdivision, it is able to generate curve interpolating some given vertices. It is more suitable for practical application. The refinement rules are derived to ensure that the eigenvalues of the refinement matrix satisfy the necessary condition of C^1continuity. Four-point interpolatory-corner cutting smooth subdivision also contains tension parameters, it is conducive to regulate the shape of limit curve.

1. Introduction

Subdivision methods are widely used in many areas including CAGD, CG and related areas. In general, subdivision scheme can be divided into two categories:interpolatory and approximate. The interpolatory subdivision interpolate all initial vertices and new vertices are inserted as linear combinations of old vertices. Repeating the process leads eventually to a smooth limit curve.For example, the classical binary four point scheme was described by Dyn et al.[1] which can achieve C^1continuous curves, and the four point scheme's properties were described by[2–4]. Approximate subdivision generates curves that approximate the control polygons.For example, Chaikin's algorithm [5] and the corner cutting subdivision[6] are two famous approximate curve subdivision schemes. Particularly, Chaikin's algorithm is a special case of the corner cutting subdivision, these algorithms actually produce curves with C^1 continuity. However, approximate subdivision and interpolatory subdivision have been proposed to improve the quality and continuity of curves shape in recent years. For example, Hassan et al. [7,8] present a ternary 4-point interpolation scheme and ternary 3-point approximate scheme that generate C^2 continuity limiting curves. Tan et al.[9] also present a new four-point shape-preserving C^3 subdivision. Li et al. first present interproximate curve subdivision in [10], which combine interpolatory and approximate subdivision.It is expected to improve behavior of the limit curves. Others about subdivision were described in [11–14].

However, in the process of curve shape structure, many surface modeling need to interpolate some given points but not all, the interpolate subdivision and approximate subdivision cannot achieve it.Now, we present a new algorithm that interpolate some given vertices which want to fixed,and approximate others.(see Fig. 1) The algorithm called four point interpolatory-corner cutting subdivision.We choose four point interpolatory and corner cutting subdivision, because they are simple and fast.

We attempt to combine the four point interpolatory and the corner cutting subdivision to construct four point interpolatory-corner cutting subdivision, but not linearly blend. Our scheme combines the two subdivision schemes in a new way that for

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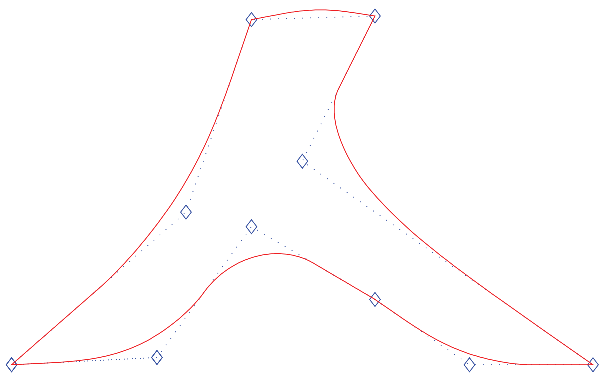

Fig. 1. A subdivision curve interpolates some control vertices and approximates the others.

a portion of the control polygon consisting of vertices required to be interpolated, the four point interpolatory subdivision is used. For a portion consisting of vertices required only to be approximated, the corner cutting subdivision is used. And for portion connecting an interpolation vertex and approximate vertex, we need new rules. The new rules are derived to assure that the eigenvalues of the refinement matrix satisfy the necessary condition of C^1 continuity. When all vertices are not needed to be interpolated, the four point interpolation-corner cutting subdivision reduces to the corner cutting subdivision. Section 2 introduce the new algorithm, Section 3 analyze the continuity of the four point interpolatory-corner cutting subdivision. The experiment shows our scheme is better than four point interpolatory subdivision and its applications in Section 4.

2. Four point interpolatory-corner cutting subdivision

This section first introduces the four point interpolatory subdivision and the corner cutting subdivision, then derived the geometric rules for the four point interpolatory-corner cutting subdivision, finally, based on the derived geometric rules, we present four point interpolation-corner cutting subdivision algorithm.

2.1. Four point interpolatory subdivision

Given a set of initial control points $P^0 = \{p_i^0 \in R^d\}_{i=1}^n$, let $P^k = \{p_i^k \in R^d\}_{i=1}^{2^k n}$ be the set of control points at level k. Then the refined polygon $P^{k+1} = \{p_i^{k+1} \in R^d\}_{i=1}^{2^{k+1} n}$ is obtained by applying the following subdivision rules :

$$\begin{cases} P_{2i}^{k+1} = p_i^k, \\ P_{2i+1}^{k+1} = \left(w + \frac{1}{2}\right)(p_i^k + p_{i+1}^k) - w(p_{i-1}^k + p_{i+2}^k), \end{cases} \tag{1}$$

where w is a tension parameter, when $0 < w < \frac{1}{8}$, the limiting curves achieve C^1 continuity. In general, $w = \frac{1}{16}$ is a popular choice, which gives the best continuity properties.

2.2. Corner cutting subdivision

Given a set of initial control points $P^0 = \{p_i^0 \in R^d\}_{i=1}^n$, let $P^k = \{p_i^k \in R^d\}_{i=1}^{2^k n}$ be the set of control points at level k. Then the refined polygon $P^{k+1} = \{p_i^{k+1} \in R^d\}_{i=1}^{2^{k+1} n}$ is obtained by applying the following subdivision rules :

$$\begin{cases} P_{2i}^{k+1} = (1 - u_i^k)p_i^k + u_i^k p_{i+1}^k, \\ P_{2i+1}^{k+1} = t_i^k p_i^k + (1 - t_i^k)p_{i+1}^k, \end{cases} \tag{2}$$

where u_i^k, t_i^k are cutting parameters, when $u_i^k > 0, t_i^k > 0, 2u_i^k + t_i^k < 1, u_i^k + 2t_i^k < 1$, the limiting curve reach C^1 continuity.

2.3. Geometric rules for four point interpolatory-corner cutting subdivision

Four point interpolatory subdivision each edge point is a linear combination of four old vertices and each vertex point is the same as an old vertex. For corner cutting subdivision, each edge point is a linear combination of two old vertices. Now we derive geometric rules for four point interpolatory-corner cutting subdivision. Our objective is to make refinement rules which have a pattern similar to the four point interpolatory and corner cutting subdivision to generate curve shapes which we want.

Given a vertex sequence $P^0 = \{p_i^0 \in R^d\}_{i=1}^n$, We classify the vertices into two categories: interpolatory and approximate. An interpolatory vertex is labeled by 'I' and remains at the same location during the refinement as in the four point interpolatory