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A Haar wavelets method of solving differential equations characterizing the dynamics of a current collection system for an electric locomotive☆

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ABSTRACT

A Haar wavelets method under certain conditions is proposed so as to numerically integrate a system of differential equations and characterize the dynamics of a current collection system for an electric locomotive. A set of Haar wavelets is employed as the basis of approximation. The *operational matrix of integration* and the *Haar Stretch Matrix* (HSM), based upon the beneficial properties of Haar wavelets, are derived to tackle the functional differential equations containing a term with a stretched argument. The unknown Haar coefficient matrix will be obtained in the generalized Lyapunov equation. The local property of Haar wavelets is applied to shorten the calculation in the task. A brief comparison of Haar wavelet with other orthogonal functions is demonstrated as well.

1. Introduction

The dynamics of the system is characterized by a system of differential equations in which one term has a stretched argument, as revealed in the investigation of the dynamics of an overhead current collection mechanism for an electric locomotive by Ockendon and Taylor [1]. The differential equations containing terms with a stretched argument λ play important roles in describing the dynamics of current collection systems for electric locomotives [1] and in describing the particulate systems [2]. Scholars [3] researched a number of aspects regarding the numerical integration of such equations through standard finite difference methods and discussed the nature of the solutions under the condition close to unity stretch, $\lambda = 1 + \epsilon$, $\epsilon \ll 1$, by means of perturbation techniques.

The functional differential equations are much more difficult to be solved than the ordinary differential equations in essence. In literature, Fox et al. [3] first obtained the solutions by his finite difference method for the case of $\lambda > 1$ and by the Lanczos method for the case of $0 < \lambda < 1$. A number of authors [4–8] made efforts to overcome the difficulties through various transform methods on the study of linear time-invariant functional equations. Haar wavelets are not only applied extensively for signal processing in communications and physics research but also proven to be a useful mathematical tool. Chen and Hsiao [9] pioneered the work in dynamic system analysis via Haar wavelets and derived a *Haar operational matrix* for the integrals of the Haar function vector. Then Hsiao [10], who first proposed a *Haar product matrix* and a *coefficient matrix*, developed the work in state analysis of linear time delayed systems via Haar wavelets. Haar transform to the solution of time-invariant functional

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differential equations is applied in this paper to take the advantages of the local property [11] and the multiplicative property of Haar wavelets wholly.

The orthogonal set of Haar wavelets [9,10,12] $h_i(t)$ is a group of square waves with magnitude of ± 1 in some intervals and zeros elsewhere. In general

$$h_n(t) = h_1(2^j t - k), \quad n = 2^j + k, \quad j \ge 0, \quad 0 \le k < 2^j, \quad n, \quad j, \quad k \in Z. \tag{1}$$

Any square integrable function $y(t)$ in the interval [0, 1] can be expanded in a Haar series with an infinite number of terms

$$y(t) = \sum_{i=0}^{\infty} c_i h_i(t), \quad i = 2^j + k, \quad j \ge 0, \quad 0 \le k < 2^j, \quad t \in [0, 1), \tag{2}$$

where the Haar coefficients

$$c_i = 2^j \int_0^1 y(t) h_i(t) dt \tag{3}$$

are determined such that the following integral square error ϵ is minimized

$$\epsilon = \int_0^1 \left[y(t) - \sum_{i=0}^{m-1} c_i h_i(t) \right]^2 dt, \quad m = 2^j, \quad j \in \{0\} \cup N \tag{4}$$

by applying the orthogonal relationship

$$\int_0^1 h_i(t) h_l(t) dt = 2^{-j} \delta_{il} = \begin{Bmatrix} 2^{-j}, & i = l = 2^j + k, \quad j \ge 0, \quad 0 \le k < 2^j \\ 0, & i \ne l \end{Bmatrix}. \tag{5}$$

Usually, the series expansion of (2) contains an infinite number of terms for a smooth $y(t)$. If $y(t)$ is piecewise constant or may be approximated as piecewise constant, then the sum in (2) may be terminated after m terms, that is

$$y(t) \approx \sum_{i=0}^{m-1} c_i h_i(t) = \tilde{c}_m^T \tilde{h}_m(t) \triangleq \hat{y}(t), t \in [0, 1), \tag{6}$$

$$\tilde{c}_m \triangleq [c_0 c_1 \cdots c_{m-1}]^T, \tag{7}$$

$$\tilde{h}_m(t) \triangleq [h_0(t) h_1(t) \cdots h_{m-1}(t)]^T. \tag{8}$$

where "T" indicates transposition, the subscript m in the parentheses denotes their dimensions, $\hat{y}(t)$ denotes the truncated sum. Let the m-square Haar matrix be defined as

$$H_{m\times m} \triangleq [\tilde{h}_m(1/(2m)) \tilde{h}_m(3/(2m)) \cdots \tilde{h}_m((2m-1)/(2m))]. \tag{9}$$

Substituting $t = 1/(2m), 3/(2m), \ldots, (2m-1)/(2m)$ into (6) yields

$$[\hat{y}(1/(2m)) \hat{y}(3/(2m)) \cdots \hat{y}((2m-1)/(2m))] = \tilde{c}_m^T H_{m\times m}. \tag{10}$$

It is obvious that

$$\tilde{c}_m^T = [\hat{y}(1/(2m)) \hat{y}(3/(2m)) \cdots \hat{y}((2m-1)/(2m))] H_{m\times m}^{-1}. \tag{11}$$

Eq. (11) is called the *forward transform*, which transforms the time function $\hat{y}(t)$ into the *coefficient vector* $\tilde{c}_m^T$, and equation (10) is called the *inverse transform*, which recovers $\hat{y}(t)$ from $\tilde{c}_m^T$. Since $H_{m\times m}$ and $H_{m\times m}^{-1}$ contain many zeros, the Haar transform is much faster than the Fourier transform and even faster than the Walsh transform.

In practical applications, a small number of terms will increase the calculation speed and save memory storage, while a large number of terms will improve the resolution. Therefore, a trade-off between the calculation speed, memory saving and the resolution should be taken in the system analysis [13].

2. Some properties of Haar wavelets

2.1. Integration of Haar wavelets

In the wavelet analysis for a dynamic system, all functions need to be transformed into Haar series. Since the differentiation of Haar wavelets always results in impulse functions which must be avoided, the integration of Haar wavelets is preferred. The integration of Haar wavelets should be expandable into Haar series with Haar coefficient matrix P [9].

$$\int_0^t \tilde{h}_m(\tau) d\tau \approx P_{m\times m} \tilde{h}_m(t), t \in [0, 1), \tag{12}$$