Applied Mathematics and Computation 265 (2015) 994–1002

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Analytical and soliton solutions: Nonlinear model of nanobioelectronics transmission lines

Muhammad Younis [a,*], Syed Tahir Raza Rizvi [b], Safdar Ali [c]

[a] Centre for Undergraduate Studies, University of the Punjab, Lahore 54590, Pakistan
[b] Department of Mathematics, COMSATS Institute of Information Technology, Lahore, Pakistan
[c] Department of Mathematics, Minhaj University, Lahore, Pakistan

ARTICLE INFO

Keywords:
Analytical solutions
Bright solitons
Dark solitons
Singular solitons
Nanobioelectronics transmission lines model

ABSTRACT

In this article, analytical solutions and different types of soliton envelopes: bright, dark and singular for the nonlinear model, namely, nanobioelectronics transmission lines have been constructed along with constrained conditions. The modified extended tanh-function method and exp-function method have been used to find analytical solutions, and while solitary wave ansatz is used to construct these soliton solutions. Additionally, the constraint conditions, for the existence of the soliton solutions are also listed.

1. Introduction

Nonlinear phenomenon is one of the fundamental behavior or situation of nature and a growing interest has been given to the propagation of nonlinear waves in the dynamical systems. Most of these systems are represented in the form of nonlinear partial differential equations (NPDEs). The study of NPDEs has become attention for scientists and researchers due to their variety of applications in electrochemistry, electromagnetic, fluid dynamics, acoustics, cosmology, astrophysics and plasma physics [1–4] *etc.* The numerous approaches have been developed or proposed by different scientists and researchers to find the soliton solutions [5–8], exact traveling wave solutions [9–13] and numerical solutions [14,15] of dynamical systems as well, in recent times.

Solitons are ubiquitous in nature, appearing in diverse system such as shallow water waves [16], DNA excitations [17], matter waves in Bose–Einstein condensates [18] and ultra-short pulses in nonlinear optics [19]. There are different types of envelope solitons, bright and dark, can propagate in nonlinear dispersive media. Compared with bright soliton which is pulse on a zero intensity background, the dark soliton appears as an intensity dip in an infinitely extended constant background.

In this article, the nonlinear model, namely, nanobioelectronics transmission lines [20] has been considered to find the analytical solutions and soliton envelopes. The analytical solutions have been constructed using the modified extended tanh-function method, while soliton envelopes: bright, dark and singular solitons have been constructed using solitary wave ansatz. The model states the possible use of microtubules as protein structure for building biomolecular nanoscale nonlinear transmission lines in the context of the polyelectrolyte character of the cytoskeletal filaments. Microtubules are major cytoskeletal proteins. They are hollow cylinders formed by protofilaments representing series of proteins known as tubulin dimers. Each dimer is an electric dipole. These diamers are in a straight position within protofilaments or in radially displaced positions pointing out of cylindrical

* Corresponding author. Tel.: +924299232068.
E-mail addresses: younis.pu@gmail.com (M. Younis), strrizvi@gmail.com (S.T.R. Rizvi), safdarali.mu@gmail.com (S. Ali).

http://dx.doi.org/10.1016/j.amc.2015.05.121
0096-3003/

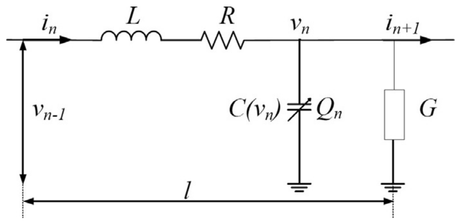

Fig. 1. An effective circuit diagram [20], for the nth ER with characteristic elements for Kirchhoff's laws.

surface. The model, for nanobioelectronics transmission lines, is read as

$$\frac{\partial^2 V}{\partial t^2} - \frac{1}{C_0 L}\frac{\partial^2 V}{\partial x^2} + \frac{RG}{C_0 L}V - \frac{l^2}{12 C_0 L}\frac{\partial^4 V}{\partial x^4} - b\frac{\partial^2 V^2}{\partial t^2} + \left(\frac{R}{L} + \frac{G}{C_0}\right)\frac{\partial V}{\partial t} - \frac{Rb}{L}\frac{\partial V^2}{\partial t} = 0. \tag{1.1}$$

Where V is the voltage in the form of the traveling wave in microtubules (MTs), C_0 represents capacitance of an electric elementary unit (EEU) of MT, L is the conductance of nano-pores, R is the resistance of an electric elementary unit of MT, G is the leakage of the conductance, l stands for the length of a polymer unit and b is the reduced factor of nonlinearity. On the basis of these estimations for the resistive components of the EEU of an MT, a typical section scheme is shown in Fig. 1. In Fig. 1, v_n represents the voltage with respect to ground while i_n represents current at node n, for more details see [20].

The rest of the article is organized as follows. In Section 2, the modified extended tanh-function method is being discussed and used to carry the analytical solutions to the transmission lines model (1.1) in Section 2.1. The Section 3, gives the overview of exp-function method and it is applied on the model in Section 3.1. The Sections 4, 5 and 6 are dealt with the soliton solutions of bright, dark and singular, respectively. The solitons are constructed using the solitary wave ansatz. In the last Section 7, the conclusion has drawn.

In the following section, the modified extended tanh-function method has been described to find the solutions.

2. The modified extended tanh-function method

In this section, the modified extended tanh-function method [21] has been discussed to obtain the analytical solutions of NPDEs. For this, we consider the NPDEs in the following way:

$$P(v, v_x, v_t, v_{xx}, \ldots) = 0, \tag{2.2}$$

where v is an unknown function and P is a polynomial of v and its partial fractional derivatives along with the involvement of higher order derivatives and nonlinear terms.

To find the exact solutions, the method can be performed using the following steps.

Step 1: First, we convert the NPDEs into nonlinear ordinary differential equations using traveling wave transformation $v(x,t) = v(\xi), \xi = x \pm ct$, which permits to reduce the above equation to an ODE of $v = v(\xi)$ in the following form

$$P(v, v', v'', v''', \ldots) = 0. \tag{2.3}$$

Step 2: Suppose that the solution of Eq. (2.2) can be expressed by the following ansatz:

$$u(\xi) = a_0 + \sum_{i=0}^{m}(a_i\phi^i + b_i\phi^{-i})^j, \tag{2.4}$$

$$\phi' = \beta + \phi^2, \tag{2.5}$$

where β is a parameter to be determined, $\phi = \phi(\xi), \phi' = \frac{d\phi}{d\xi}$.

Step 3: The homogeneous balance can be used, to determine the positive integer m, between the highest order derivatives and the nonlinear terms appearing in Eq. (2.3).

Step 4: After the substitution of Eqs. (2.4) and (2.5) into Eq. (2.3), we collect all the terms with the same order of ϕ^j together. Equate each coefficient of the polynomials of ϕ^j to zero, yields the set of algebraic equations. After solving the system of algebraic equations, fortunately, the Riccati equation admits several types of following solutions:

(i) If $\beta < 0$

$$\phi = -\sqrt{-\beta}\tanh(\sqrt{-\beta}\xi), \quad or \quad \phi = -\sqrt{-\beta}\coth(\sqrt{-\beta}\xi), \tag{2.6}$$

it depends on initial conditions. (ii) If $\beta > 0$

$$\phi = \sqrt{\beta}\tan(\sqrt{\beta}\xi), \quad or \quad \phi = -\sqrt{\beta}\cot(\sqrt{\beta}\xi), \tag{2.7}$$

it depends on initial conditions. (iii) If $\beta = 0$

$$\phi = \frac{-1}{\xi}. \tag{2.8}$$