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Convergence radius of Halley's method for multiple roots under center-Hölder continuous condition

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ABSTRACT

Recently, a new treatment based on Taylor's expansion to give the estimate of the convergence radius of iterative method for multiple roots has been presented. It has been successfully applied to enlarge the convergence radius of the modified Newton's method and Osada's method for multiple roots. This paper re-investigates the convergence radius of Halley's method under the condition that the derivative $f^{(m+1)}$ of function f satisfies the center-Hölder continuous condition. We show that our result can be obtained under much weaker condition and has a wider range of application than that given by Bi et. al.(2011) [21].

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1. Introduction

Solving non-linear equations is a common and important problem in science and engineering. In this study, we consider the iterative method for finding a root x^* of multiplicity m of a nonlinear equation $f(x) = 0$ on an open interval $D \subseteq R$, i.e., $f^{(i)}(x^*) = 0, i = 0, 1, \dots, m-1$ and $f^{(m)}(x^*) \neq 0$.

It is known that, the modified Newton's method for multiple roots, given by Schröder[1], is quadratically convergent and defined by

$$x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}. \tag{1}$$

The main goal and motivation in constructing iterative methods for solving nonlinear equations are to give higher order iterative methods with minimal computational cost. The Kung–Traub conjecture[2] suggests that an effective way to improve the order of convergence of iteration is to use more information of function f. For example, using multistep iterative approach, a lot of higher-order iterative methods have been presented, see [3–11] and reference therein. Another efficient way is using single-step iteration, but in this case, the second or higher–order derivative of f should be needed.

For example, in [12], Traub has presented the following cubically convergent iterative method

$$x_{n+1} = x_n - \frac{m(3-m)}{2}\frac{f(x_n)}{f'(x_n)} - \frac{m^2}{2}\frac{f(x_n)^2 f''(x_n)}{f'(x_n)^3},$$

which can be viewed as an extension of Chebyshev's method.

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Another famous third order method for multiple roots is Halley's method, presented by Hansen and Patrick[13]

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{m+1}{2m} f'(x_n) - \frac{f(x_n) f''(x_n)}{2 f'(x_n)}}. \tag{2}$$

Also using the second derivative, Osada[14] gives another cubically convergent iterative method for multiple roots

$$x_{n+1} = x_n - \frac{1}{2} m(m+1) \frac{f(x_n)}{f'(x_n)} + \frac{1}{2}(m-1)^2 \frac{f'(x_n)}{f''(x_n)}. \tag{3}$$

Although more and more iterative methods for multiple roots have been presented, these results show that if the initial guess x_0 is sufficiently close to the root x^* of the function involved, the sequence $\{x_n\}$ generated by the method is well defined, and converges to x^*. But, how close to the root x^* the initial guess x_0 should be? These local results give no information on the radius of the convergence ball for the corresponding method. In fact, for the iterative methods for simple roots, many results on the radius of convergence ball have been studied. However, there are few results on the iterative methods for multiple roots. Until recently, Ren and Argyros first give an estimate of the convergence radius of the modified Newton's method (1) in [15], assuming that the function f satisfies the Hölder continuous condition

$$|f^{(m)}(x^*)^{-1}(f^{(m)}(x) - f^{(m)}(y))| \leq K|x-y|^p, \quad \forall x, y \in D,\ K > 0, \tag{4}$$

and the center-Hölder continuous condition

$$|f^{(m)}(x^*)^{-1}(f^{(m)}(x) - f^{(m)}(x^*))| \leq K_0|x-x^*|^p,\ \forall x, y \in D,\ K_0 > 0, \tag{5}$$

where $0 < p \leq 1$, K and K_0 are positive constants.

Remark 1. It should be pointed out that $K_0 < K$ and the radio $\frac{K}{K_0}$ can be arbitrarily large[16]. What is more, the constant $K > 0$ may not exist (see Example 3 or Example 4.3 in [15]). So, the center-Hölder continuous condition (5) is much weaker than the Hölder continuous condition (4).

Their work is a milestone, which not only extends the results in [17–19], but also shows us an approach to deal with the local convergence analysis of the iterative methods for multiple roots. The key idea of them can be summarized as

1. Higher-order derivative
2. Higher-order divided difference
3. Multiple integral
4. Integral inequality

This approach has been successfully applied to give the convergence radius of Osada's method[20] and Halley's method[21]. Under the Hölder continuous condition

$$|f^{(m)}(x^*)^{-1}(f^{(m+1)}(x) - f^{(m+1)}(y))| \leq K|x-y|^p, \quad \forall x, y \in D \subseteq R,\ K > 0,$$

and then bounded condition

$$|f^{(m)}(x^*)^{-1} f^{(m+1)}(x)| \leq M, \qquad \text{for some} \quad M > 0.$$

Bi et al. give the estimate of the convergence radius of Halley's method (2) as the minimum positive zero r_* of the function

$$\begin{aligned} h(r) = {} & 2mm!MKr^{p+2} + (m-1)\prod_{i=1}^{m+1}(m+p+2-i)K^2r^2 - 2m(m+1)!Mr^{p+1} \\ & -m(2m+1)\prod_{i=1}^{m+1}(m+p+2-i)Kr + m^2(m+1)\prod_{i=1}^{m+1}(m+p+2-i). \end{aligned} \tag{6}$$

Different from the approach given by Ren and Argyros in [15], recently, another treatment, based on Taylor expansion, has been presented by Zhou et. al.[22], which can be outlined as

1. Higher-order derivative
2. Taylor expansion with integral form remainder
3. Integral inequality

Obviously, this approach is simpler than Ren and Argyros. Above all, Zhou et. al. have shown that even under the weaker condition (the center-Hölder continuous condition (5)), better results can be obtained for the modified Newton's method (1) and Osada's method (3).

So, in this work, we reconsider the local convergence of Halley's method (2) by Taylor expansion under the condition that $f^{(m+1)}$ satisfies the center-Hölder continuous condition

$$|f^{(m)}(x^*)^{-1}(f^{(m+1)}(x) - f^{(m+1)}(x^*))| \leq K_0|x-x^*|^p,\ \forall x \in D,\ K_0 > 0. \tag{7}$$

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