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# New admissibility analysis for discrete singular systems with time-varying delay[☆]

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ABSTRACT

In this paper, the issue of admissibility for discrete singular systems with time-varying delay is addressed. The main contribution of this paper is to reduce the conservatism of the considered system by adopting an improved reciprocally convex combination approach to bound the forward difference of the double summation term, and utilizing the reciprocally convex combination approach to bound the forward difference of the triple summation term in the Lyapunov function. Without employing decomposition and equivalent transformation of the considered system, a strict delay-dependent linear matrix inequality (LMI) criterion is built to guarantee the considered system to be regular, causal and stable. A numerical example is given to illustrate the effectiveness of this proposed method.



## 1. Introduction

Singular systems have appealed many researchers' interest in the past several decades, since they have widespread applications in practical engineering, such as power systems, large scale electrical circuit systems, economic systems, robot control systems [1,15,26]. A lot of results on standard state-space systems have already been extended to singular systems [7,27]. However, the study for singular systems is much more complicated than that for the standard state-space systems, because not only the stability should be considered, but also the regularity and absence of impulses (for continuous singular systems) or causality (for discrete singular systems) [25]. The latter two issues do not show up in standard state-space systems [13]. In addition, it is well known that time-delay exists widely in many natural systems and is the main cause of instability and poor performances of dynamic systems [17,21]. Consequently, a lot of studies of time-delay systems have been carried out [18,28]. With the advantage of the linear matrix inequality (LMI) approach, a number of important results have been developed for all kind of time-delay systems [6,10,11].

There have been a lot of works done on the admissibility of singular systems with time-varying delay and many criteria have been established in terms of the LMI approach. For discrete-time singular systems with constant time-delay, the delay partitioning technique is applied to establish strict LMI sufficient criteria in [5]. When time-varying delay appears, the restricted system equivalent transformation is utilized in [14] to propose a delay-dependent LMI condition guaranteeing the discrete-time singular systems to be admissible. However, in [14], the established condition is in terms of the system matrices of the

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decomposed systems, which makes the stability analysis procedure indirect and complicated. A sufficient admissibility criterion is given in [3] without resorting to the decomposition and equivalent transformation of the considered system. The problem of delay-dependent robust stability for uncertain discrete-time singular systems with time-varying delay is investigated in [2], in which the results are proposed in terms of strict LMIs without decomposition of system matrices, but the regularity and causality of the system are not considered. In [22], the problem of admissibility for discrete-time singular systems with discrete and distributed delays is taken into consideration by taking advantage of the delay partitioning technique. Recently, the reciprocally convex combination method which significantly reduces the conservatism of the results about time-delay systems is proposed in [16]. The reciprocally convex combination approach has been utilized to analyze the dissipativity problem in [4,19,23]. An improved version of the reciprocally convex combination method is represented in [9] by adding three new equalities where triple-integral technique derived from continuous-time delay systems is successfully used in discrete-time delay systems. However, the reciprocally convex combination approach has not been used to bound the forward difference of a triple-summation term of Lyapunov function in the aforementioned references. Generally speaking, the reciprocally convex combination approach has a better performance on bounding the forward difference of a double-summation term of Lyapunov function than that by using the Jensen inequality [20]. This motivates us to use this idea to improve the existing results on admissibility of discrete-time singular systems with time-varying delay.

This paper investigates the problem of admissibility analysis for discrete-time singular system with time-varying delay. By employing the reciprocally convex combination approach to bound the forward difference of a triple-summation term, a sufficient criterion is presented in terms of LMIs to guarantee the considered system to be regular, causal and stable. Finally, a numerical example is exhibited to illustrate the effectiveness and the reduced conservatism of the proposed result.

*Notation:* $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ donate the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. $P > 0$ ($\geq 0$) means that $P$ is a real symmetric and positive definite (semi-definite) matrix. $I$ and 0 represent the identity matrix and a zero matrix with compatible dimensions, respectively. The superscript $T$ represents the transpose of the matrices. $\star$ stands for the symmetric terms in a symmetric matrix and sym$(A)$ is defined as $A + A^T$. Given a sequence $x_a, x_{a+1}, \dots, x_b$, its sum, written as $\sum_{i=a}^{b} x_i$, is zero, if $b < a$.

## 2. Problem formulation

Consider discrete-time singular systems with time-varying delay described by

$$\begin{aligned} Ex(k+1) &= Ax(k) + A_d x(k-d(k)) \\ x(k) &= \phi(k), \quad k \in [-d_2, 0] \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector and $d(k)$ is a time-varying delay satisfying $0 \leq d_1 \leq d(k) \leq d_2$, where $d_1$ and $d_2$ are prescribed nonnegative integers representing the lower and upper bounds of the time delay, respectively. $\phi(k)$ is the compatible initial condition. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular, and it is assumed that rank$(E) = r \leq n$. $A$ and $A_d$ are known real constant matrices with appropriate dimensions.

The following definitions and lemmas will be used in the proof of the main results:

**Definition 1.** [1]

1. The pair $(E, A)$ is said to be regular, if $\det(zE - A)$ is not identically zero.
2. The pair $(E, A)$ is said to be causal, if $\deg(\det(zE - A)) = \mathrm{rank}(E)$.

**Definition 2.** [24] The singular system in (1) is said to be stable if for any scalar $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$ such that, for any compatible initial conditions $\phi(k)$ satisfying $\sup_{-d_2 \leq k \leq 0} \|\phi(k)\| \leq \delta(\varepsilon)$, the solution $x(k)$ of system (1) satisfies $\|x(k)\| \leq \varepsilon$ for any $k \geq 0$, moreover $\lim_{k\to\infty} x(k) = 0$.

**Definition 3.** [14]

1. The singular system in (1) is said to be regular and causal, if the pairs $(E, A)$ is regular and causal.
2. The singular system in (1) is said to be admissible if it is regular, causal and stable.

**Lemma 1.** *[16] Let $f_1, f_2, \dots, f_N : \mathbb{R}^m \to \mathbb{R}$ have positive values in an open subset $D$ of $\mathbb{R}^m$. Then the reciprocally convex combination of $f_i$ over $D$ satisfies*

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(k) = \sum_i f_i(k) + \max_{g_{i,j}(k)} \sum_{i \neq j} g_{i,j}(k) \tag{2}$$

$$\textit{subject to} \left\{ g_{i,j} : \mathbb{R}^m \to \mathbb{R}, g_{j,i}(k) = g_{i,j}(k), \begin{bmatrix} f_i(k) & g_{j,i}(k) \\ g_{i,j}(k) & f_j(k) \end{bmatrix} \geq 0 \right\}$$

**Lemma 2.** *For any matrix $M > 0$, integers $a < b \leq c$, vector function $x(i) : [k+a, k+c-1] \to \mathbb{R}^n$, there hold*

$$-(b-a) \sum_{i=a}^{b-1} x^T(i) M x(i) \leq -\left( \sum_{i=a}^{b-1} x(i) \right)^T M \left( \sum_{i=a}^{b-1} x(i) \right) \tag{3}$$