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Solving a class of linear nonlocal boundary value problems using the reproducing kernel

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A B S T R A C T

Recently, the reproducing kernel Hilbert space methods(RKHSM) (see Wang et al (2011) [2]; Lin and Lin (2010) [3]; Wu and Li (2011) [4]; Zhou et al. (2009) [6]; Jiang and Chen (2014) [7]; Wang et al. (2010) [8]; Du and Cui (2008) [9]; Akram et al. (2013) [10]; Lü and Cui (2010) [11]; Wang et al. (2008) [12]; Yao and Lin (2009) [13]; Geng et al. (2014) [14] ; Arqub et al. (2013) [15]) emerged one after the other. But, a lot of difficult work should be done to deal with multi-point boundary value problems(BVPs). Our work is aimed at giving a new reproducing kernel method for multi-point BVPs. We do not put the homogenization conditions into the reproducing kernel space which can avoid to compute the reproducing kernel satisfying boundary conditions and the orthogonal system. Three numerical examples are studied to demonstrate the accuracy of the present method. Results obtained by our method indicate that new algorithm has the following advantages: small computational work, fast convergence speed and high precision.

1. Introduction

In previous works, the reproducing kernel method must homogenize boundary value conditions, and after homogenization of these conditions, authors put these homogenization conditions into the reproducing kernel space and compute a reproducing kernel satisfying these homogenization conditions (see Refs. [2–4,6–15]). This paper is concerned with a new algorithm for giving the approximate solution of a class of multi-point boundary value problems in the reproducing kernel space. We avoid to compute the reproducing kernel satisfying boundary conditions and the orthogonal system. Three numerical examples are given to demonstrate the efficiency of the present method. The present method when compared with the others methods, reveals that it is more effective and convenient.

As we know, boundary value problems(BVPs) arise in many fields(see Refs. [16–24]). In [1], Henderson and Kunkel proved the uniqueness of solutions for the following linear differential equations with nonlocal boundary conditions:

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$$\begin{cases} u^{(m)}(x) + \sum_{i=0}^{m-1} a_i(x)u^{(i)}(x) = f(x) \\ u^{(i-1)}(x_j) = b_{ij}, 1 \le i \le m_j, 1 \le j \le k \\ u(x_{k+1}) - u(x_{k+2}) = b_m \end{cases} \tag{1}$$

where $a_i(x) \in C[a, b]$, m_j are positive integers such that $\sum_{i=1}^{k} m_i = m - 1$, $a < x_1 < x_2 < \cdots < x_{k+2} < b$, b_m, b_{ij} are real numbers.

In [3,4], Lin and Wu use the reproducing kernel to solve the following boundary value problems(BVPs).

$$\begin{cases} u^{(4)}(x) + \sum_{i=0}^{3} a_i(x)u^{(i)}(x) = f(x) \\ u^{(i-1)}(\xi_1) = b_i, 1 \le i \le 3, u(\xi_2) - u(\xi_3) = b_4 \end{cases} \tag{2}$$

In this work, we give a new numerical algorithm to solve (2) in the reproducing kernel, We don't put the homogenization conditions into the reproducing kernel space. We avoid to compute the reproducing kernel satisfying boundary conditions and the orthogonal system. Three numerical examples are given to demonstrate the efficiency of the present method. The present method compared with the others methods, reveals that it is more effective and convenient.

2. A new algorithm

We define the inner product space $W_2^5[0, 1] = \{u | u, u', \ldots u^{(4)}$ is a scalar absolutely continuous function, $u^{(5)} \in L^2[0, 1]\}$. The inner product in $W_2^5[0, 1]$ is given by

$$\langle u(x), v(x)\rangle = \sum_{i=0}^{5} u^{(i)}(0)v^{(i)}(0) + \int_0^1 u^{(5)}(t)v^{(5)}(t)dt \tag{3}$$

The space $W_2^5[0, 1]$ is a reproducing kernel space, and its reproducing kernel is

$$K_x(y) = \begin{cases} \frac{1}{576}\left(576 + \frac{x^9}{630} + 576xy - \frac{x^8y}{70} + 144x^2y^2 + \frac{2x^7y^2}{35} + 16x^3y^3 - \frac{2x^6y^3}{15} + x^4y^4 + \frac{x^5y^4}{5}\right), x \le y \\ \frac{1}{576}\left(576 + \frac{y^9}{630} + 576xy - \frac{y^8x}{70} + 144x^2y^2 + \frac{2y^7x^2}{35} + 16x^3y^3 - \frac{2y^6x^3}{15} + x^4y^4 + \frac{y^5x^4}{5}\right), y < x \end{cases} \tag{4}$$

Let

$$L = [A, B_i]^T, F(x) = [f(x), b_i]^T$$

where

$$Au = u^{(4)}(x) + \sum_{i=0}^{3} a_i(x)u^{(i)}(x)$$

$$B_iu = u^{(i-1)}(\xi_1), i = 1, 2, 3, B_4u = u(\xi_2) - u(\xi_3))$$

Eq. (2) can be transformed into following form in $W_2^5[0, 1]$:

$$(Au)(x) = F(x) \tag{5}$$

Lemma 2.1. *Let* $\psi_j(x) = (A_yK_x(y))(x_j)$, $\psi^{\{i\}}(x) = B_iK_x(y)$, $(i = 1, 2, 3, 4)$, *where*

$$(A_yK_x(y))(x_j) = \frac{\partial^4 K_x(y)}{\partial y^4} + \sum_{i=0}^{3} a_i(x)\frac{\partial^i K_x(y)}{\partial y^i}|_{y=x_j}, j = 1, 2, \ldots,$$

$$B_iK_x(y) = \frac{\partial^{(i-1)}K_x(y)}{\partial y^{(i-1)}}|_{y=\xi_1}, (i = 1, 2, 3), B_4K_x(y) = K_x(\xi_2) - K_x(\xi_3)$$

If A^{-1} *is existent and* $\{x_j\}_{j=1}^{\infty}$ *is distinct points dense in* $[0, 1]$, $b_i \ne 0$, $(1 \le i \le 3, 1 \le j \le k)$, *then* $\{\psi_j(x)\}_{j=1}^{\infty} \bigcup \{\psi^{\{i\}}(x)\}_{i=1}^{4}$, *is a complete function system in* $W_2^5[0, 1]$.

Proof. Since $b_i \ne 0 (1 \le i \le 4)$. For each fixed $u(x) \in W_2^5[0, 1]$, if $\langle u(x), \psi_j(x)\rangle = 0$, then

$$\langle u(x), \psi_j(x)\rangle = (A_y\langle u(x), K_x(y)\rangle)(x_j) = (A_yu(y))(x_j) = 0. \tag{6}$$

Taking into account the density of $\{x_j\}_{j=1}^{\infty}$, it results in $(A_yu(y))(x) = 0$. It follows that $u(x) \equiv 0$ from the existence of A^{-1}. □

Lemma 2.2. *If* $\{x_i\}_{i=1}^{n}$ *is distinct dense on* $[0, 1]$, *then* $\{\psi_j(x)\}_{j=1}^{n} \bigcup \{\psi^{\{i\}}(x)\}_{i=1}^{4}$ *is linearly independent in* $W_2^5[0, 1]$.