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New iterative technique for solving nonlinear equations

Muhammad Aslam Noor [a,*], Muhammad Waseem [b], Khalida Inayat Noor [a], Muhammad Aamir Ali [b]

[a] Department of Mathematics, COMSATS Institute of Information Technology, Park Road, Islamabad, Pakistan
[b] Department of Mathematics, COMSATS Institute of Information Technology, Sahiwal, Pakistan

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ABSTRACT

Various problems of pure and applied sciences can be studied in the unified frame work of the nonlinear equations. In this paper, a new family of iterative methods for solving nonlinear equations is developed by using a new decomposition technique. The convergence of the proposed methods is proved. It is shown that the new family contains several well-known iterative methods as special case. For the implementation and performance of the new methods, a nonlinear equation arising in the population model and another one arising in the motion of a particle on inclined plane is solved.

1. Introduction

In recent years, much attention has been specified to build up several iterative methods for solving nonlinear equations, see [1–15] and the references therein. Abbasbandy [1] and Chun [5] have used Adomian decomposition method [2] and its different modifications to find out the solution of nonlinear equations. In this work, a new decomposition technique [3] which is quite different from Adomian decomposition method [2], is used to develop a family of iterative methods for solving the nonlinear equations.

We would like to point out that in the implementation of the Adomian decomposition method; one has to calculate the derivatives of the so-called Adomian polynomials which it is difficulty of Adomian method. To overcome the disadvantage of Adomian method, the decomposition of Bhalekar and Daftardar-Gejji [3], as developed by Noor and Noor [9], is used in this work. The said decomposition technique [3] does not occupy the high-order differentials of the function and is very simple as compared to the Adomian method. He [8] has suggested that the nonlinear equations can be written as a coupled system of equations. Noor et al. [12–14] have used the idea of coupled system of equations with the decomposition technique of Bhalekar and Daftardar-Gejji [3], to develop several iterative methods to solve the nonlinear equations.

In this work, decomposition method [3] elegantly combined with the coupled system of equations, is used to develop new family of iterative methods. It is shown that the proposed family of iterative methods restrains several well-known iterative techniques as special case. The convergence of the suggested family of method is also proved. Some numerical examples including the population model and problem of the motion of a particle on inclined plane, are solved to illustrate the efficiency and performance of proposed iterative methods.

* Corresponding author. Tel.: +92 3454027532.
E-mail addresses: noormaslam@hotmail.com, noormaslam@gmail.com (M.A. Noor), sattarmwaseem@gmail.com (M. Waseem), khalidanoor@hotmail.com (K.I. Noor), mahr.muhammad.aamir@gmail.com (M.A. Ali).

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2. Iterative methods

Consider the nonlinear equation

$$f(x) = 0 \tag{1}$$

Using the quadrature formula and fundamental law of calculus the function $f(x)$ can be approximated as:

$$f(x) \approx f(\gamma) + \left[\sum_{i=1}^{p} w_i f'(\gamma + \tau_i(x-\gamma))\right](x-\gamma).$$

Assume that α is a simple root of nonlinear Eq. (1) and γ is an initial guess sufficiently close to α. Using the idea of Noor et al. [12,13], we can rewrite the nonlinear Eq. (1) as coupled system (see also [8]):

$$f(\gamma) + \left[\sum_{i=1}^{p} w_i f'(\gamma + \tau_i(x-\gamma))\right](x-\gamma) + g(x) = 0, \tag{2}$$

$$g(x) = f(x) - f(\gamma) - \left[\sum_{i=1}^{p} w_i f'(\gamma + \tau_i(x-\gamma))\right](x-\gamma), \tag{3}$$

where τ_i are knots, in [0,1] and w_i are the weights which verify the consistency condition

$$\sum_{i=1}^{p} w_i = 1, \tag{4}$$

and γ is initial approximation for a zero of (1). Eq. (2) can be rewritten in the following form:

$$x = \gamma - \frac{f(\gamma) + g(x)}{\sum_{i=1}^{p} f'(\gamma + \tau_i(x-\gamma))} = c + N(x), \tag{5}$$

where

$$c = \gamma, \tag{6}$$

and

$$N(x) = -\frac{f(\gamma) + g(x)}{\sum_{i=1}^{p} f'(\gamma + \tau_i(x-\gamma))}. \tag{7}$$

Here $N(x)$ is a nonlinear function. To decompose $N(x)$, we use a decomposition technique, which is mainly due to Daftardar-Gejji and Jafari [3,6]. In their work [3,6], the authors have developed the said decomposition technique to solve the nonlinear functional equations. But in this paper, we develop the same decomposition technique to suggest the iterative algorithms for nonlinear equations of the type $f(x) = 0$. This decomposition of the nonlinear function $N(x)$ is quite different from Adomian decomposition. The main idea of this technique is to find the solution in the following series form:

$$x = \sum_{i=0}^{\infty} x_i. \tag{8}$$

The nonlinear operator N can be decomposed as:

$$N(x) = N(x_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} x_j\right) - N\left(\sum_{j=0}^{i-1} x_j\right)\right\}. \tag{9}$$

Substituting (8) and (9) in (5) yields

$$\sum_{i=0}^{\infty} x_i = c + N(x_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} x_j\right) - N\left(\sum_{j=0}^{i-1} x_j\right)\right\}. \tag{10}$$

Thus, we have the following iterative scheme:

$$\begin{aligned} x_0 &= c, \\ x_1 &= N(x_0), \\ x_2 &= N(x_0 + x_1) - N(x_0), \\ &\vdots \\ x_{m+1} &= N\left(\sum_{j=0}^{m} x_j\right) - N\left(\sum_{j=0}^{m-1} x_j\right), \quad m = 1, 2, \ldots. \end{aligned} \tag{11}$$