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Delay-dependent state feedback stabilization for a networked control model with two additive input delays

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ABSTRACT

This paper is centered on delay-dependent state feedback stabilization for a networked control model with two additive input delays. Firstly delay-dependent stability is investigated. By splitting the whole delay interval into subintervals according to the delays, a Lyapunov functional is constructed. To reduce conservatism we handle the Lyapunov functional in two ways. More specifically, we take the Lyapunov functional as a whole to examine its positive definite, rather than restrict each term of it to positive definite as usual. In addition, when estimating the derivative of the Lyapunov functional, we manage to get a fairly tighter upper bound by introducing different slack variables for the different subintervals. The resulting stability results turn out dependent on the two delays separately, and less conservative than some existing ones. Then, based on the stability results state feedback stabilization is studied. Delay-dependent conditions are formulated for the controller such that the closed-loop system is asymptotically stable. Finally examples are given to show the less conservatism of the stability results and the effectiveness of the proposed stabilization method.

1. Introduction

For years systems with time delays have received considerable attention since they are often encountered in various practical systems, such as engineering systems, biology, economics, neural networks, networked control systems and other ones [1–6]. Since time-delay is frequently the main cause of oscillation, divergence or instability, considerable effort has been devoted to stability for systems with time delays. According to whether stability criteria include the information of the delay, they are divided into two classes: delay-independent stability criteria and delay-dependent ones. It is well known that delay-independent stability criteria tend to be more conservative especially for small size delays. More attention has been paid to delay-dependent stability. For delay-dependent stability results, we refer readers to [7–13]. Among these papers, [11–13] were of systems with interval time-varying delay. It is worth noting Markov jump systems with delays have wide applications in industries. For this kind of systems, readers are referred to recently published results [22–24].

On the other hand, networked control systems have been receiving great attention these years due to their advantages in low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. It is well known that the transmission delay and the data packet dropout are two fundamental issues in networked control systems. The transmission delay generally includes the sensor-to-control delay and the control-to-actuator delay. In most of existing papers the sensor-to-control delay and the control-to-actuator delay were combined into one state delay, while the data packet dropouts were modeled as delays and absorbed by the state delay, thus formulating networked control systems as systems with one state

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delay. Among recently reported results based on this modeling idea, to mention a few, H^∞ control co-design problems were addressed for networked control systems in [3], while the event-triggered control for networked control systems was studied via dynamic output feedback controllers [4]. Note that the sensor-to-control delay and the control-to-actuator delay are different in nature because of the network transmission conditions. The transmission delay and the data packet dropout also have different properties. It is not rational to lump them into one input delay. In this paper, to study networked control systems we adopt the model with additive input delays. For brevity the model with two additive input delays will be employed to address state feedback stabilization for networked control systems. Since stability is the precondition for a networked control system to work, and an unstable networked control system cannot perform the task expected. In the following we focus on the stability for a networked control system with two additive input delays. When the physical plant is a linear system and the controller is a linear state-feedback one with a controller gain K, the networked control system takes the following form:

$$\dot{x}(t) = Ax(t) + BKx(t - d_1(t) - d_2(t)) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, A and B are known real constant matrices, $d_1(t)$, $d_2(t)$ are two input delays. A nature assumption on the two delays is made

$$0 \le d_1(t) \le h_1, \quad 0 \le d_2(t) \le h_2 \tag{2}$$

and

$$\dot{d}_1(t) \le \mu_1, \quad \dot{d}_2(t) \le \mu_2 \tag{3}$$

For this kind of system stability analysis was initially conducted in [14], and a delay-dependent stability criterion was obtained. A following up investigation was given in [15] by constructing a new Lyapunov functional. Recently stability for system (1)–(3) was further studied in [16–18]. In [16] slack variables were introduced to obtain alternative results, while in [17] without introducing a slack variable a new stability result was given using the convex polyhedron technique. In [18], the author employed a new integral inequality to derive an improved stability result. However, in [14–18], it is overlooked inadvertently that the positive definite of a Lyapunov functional does not necessarily imply that of each term of it, and the conditions given in those papers was still conservative for the positive definite of a Lyapunov functional. So there is still room to improve in existing papers.

In this paper we first consider delay-dependent stability for system (1). By separating the whole delay interval into subintervals in terms of the delays we construct a new Lyapunov functional that employs the information of the marginally delayed state $x(t - h_1)$ as well as $x(t - h)$. Based on the observation that the positive definite of a Lyapunov functional does not necessarily imply that of each term of it, we take the Lyapunov functional as a whole to examine its positive definite. On the other hand, when bounding the derivative of the Lyapunov functional, we introduce different slack variables for different subintervals, thus obtaining a fairly tighter upper bound. The stability results obtained are dependent on both of the two delays and less conservative than some existing ones. Then we apply the stability results to state feedback stabilization problem, which is to determine delay-dependent condition for the state feedback controller gain K such that system (1) is asymptotically stable. When the condition is feasible the controller gain is computed.

Throughout this paper the superscript 'T' stands for matrix transposition. I refers to an identity matrix with appropriate dimensions. E_i stands for the ith row of $diag\{I_n, I_n, I_n, I_n, I_n\}$ and $E_{ij} = E_i - E_j (i, j = 1, 2, 3, 4, 5)$. For real symmetric matrices X and Y, the notation $X > Y$ means that the matrix $X - Y$ is positive definite. The $X \ge Y$ follows similarly. The symmetric term in a matrix is denoted by $*$.

To end this section, a lemma is given, which will play an important role in deriving our results.

Lemma 1. [19]: *For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds*

$$\left(\int_0^\gamma \omega(s)ds\right)^T M \left(\int_0^\gamma \omega(s)ds\right) \le \gamma \left(\int_0^\gamma \omega(s)^T M\omega(s)ds\right)$$

2. Stability analysis

In the following we consider the stability analysis problem. Specifically, given K we investigate the condition for system (1) to be asymptotically stable. To solve the problem, one routine approach is to lump $d_1(t)$ and $d_2(t)$ into one delay

$$d(t) = d_1(t) + d_2(t) \tag{4}$$

and change system (1) into

$$\dot{x}(t) = Ax(t) + BKx(t - d(t)) \tag{5}$$

where $0 \le d(t) \le h$, $\dot{d}(t) \le \mu$ with

$$h = h_1 + h_2 \tag{6}$$

$$\mu = \mu_1 + \mu_2 \tag{7}$$

However, this treatment is not suitable. On the one hand, from an engineering point of view, the two delays may have different properties, it is not appropriate to lump them together. On the other hand, from a mathematical point of view it is very