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Monotone iterative solutions for nonlinear fractional differential systems with deviating arguments ☆

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ABSTRACT

By means of the monotone iterative technique, we consider the nonlinear fractional differential systems with deviating arguments, introduce two well-defined monotone sequences of solutions which converge uniformly to the solutions of the systems, and then the existence result of solution for the systems is established. A numerical iterative scheme is introduced to obtain an accurate approximate solution for the systems. As an application, an example is presented to demonstrate the accuracy and efficiency of the new approach.

1. Introduction

We are interested in the existence and uniqueness of solution of the following nonlinear fractional order differential systems with deviating arguments

$$\begin{cases} D^{\mu}v(t) = f(t, v(t), v(\theta(t)), w(t), w(\theta(t))), & t \in (0, 1], \\ D^{\mu}w(t) = g(t, w(t), w(\theta(t)), v(t), v(\theta(t))), & t \in (0, 1], \\ t^{1-\mu}v(t)|_{t=0} = x_0, \quad t^{1-\mu}w(t)|_{t=0} = y_0, \end{cases} \tag{1.1}$$

where $0 < \mu \le 1$, $f, g \in C([0, 1] \times R \times R \times R \times R, R)$, $\theta \in C([0, 1], [0, 1])$, $x_0, y_0 \in R$ and $x_0 \le y_0$, D^{μ} is the Riemann–Liouville fractional derivative of order μ.

It is well known that the differential equations with fractional order are generalization of ordinary differential equations to non-integer order, it occurs more frequently in different research areas and engineering, such as physics, control of dynamical systems, chemistry etc. We also remark that several kinds of fractional derivatives were introduced to investigate the fractional differential equation, see for example [8,11] and references therein.

Roughly speaking, it is a difficult task to give exact solutions for fractional differential equations. Recently, there are a number of numerical and analytical techniques to concerned with such problems, for instance, the homotopy analysis method and the Adomian decomposition method have been discussed the fractional differential equations, such as [2,3,9,10]. On the other hand, the monotone iterative technique, combined with the method of lower and upper solutions was introduced to study the problems [1,4–7,13,14], In [1,5], the authors used the method of lower and upper solutions to investigate the existence of solutions for a class of fractional initial value problems. In [13] the author considered fractional boundary value problem and proved the existence of solution. In [4] the authors introduce two well-defined monotone sequences of lower and upper solutions which

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converge uniformly to the actual solution of the problem. In [14] the author discusses the monotone iterative technique for boundary value problems of a nonlinear fractional differential equation with deviating arguments. Very recently, in [6,7], we also investigated the monotone iterative solutions for nonlinear boundary value problems of fractional differential equation with deviating arguments or with no deviating arguments. Specially, here it is worth mentioning, that Wang et al. [15] discuss the existence results and the monotone iterative technique for systems of nonlinear fractional differential equations with no deviating arguments for (1.1). To the best of our knowledge, no paper can be found in the literature when the above method is applied to systems (1.1).

Being directly inspired by [15], the purpose of this paper is to study the monotone iterative solutions for nonlinear fractional differential systems with deviating arguments. We introduce two well-defined monotone sequences of solutions which converge uniformly to the solutions of the systems. And also give an algorithm to construct two monotone sequences of solutions. Moreover, the constructed sequences are proved to converge uniformly to the solutions of the systems.

The paper is organized as follows: Preliminaries are in Section 2. Then in Section 3, we construct the monotone sequences of solutions and prove their uniform convergence to the solutions of the systems. Finally, in Section 4, we establish the numerical approach employed to obtain accurate numerical solutions, and an example is presented to demonstrate the accuracy and efficiency of the new approach.

2. Preliminaries

In this section, we deduce some preliminary results which will be needed in the next section.

For $0 < \mu < 1$, the Riemann–Liouville fractional derivative of order μ is defined by (see [8,11])

$$D^{\mu}h(t) = \frac{1}{\Gamma(1-\mu)}\frac{d}{dt}\int_0^t (t-s)^{-\mu}h(s)\,ds = \frac{d}{dt}I^{1-\mu}h(t),$$

here

$$\int_0^t (t-s)^{-\mu}h(s)\,ds = I^{1-\mu}h(t)$$

is Riemann–Liouville fractional integral of order $1-\mu$ (see [8,11]).

Setting

$$C_{1-\mu}([0,1]) = \{u \in C(0,1] :\ t^{1-\mu}u \in C([0,1])\}.$$

Lemma 2.1. *([6]) Let $M_0 > 0$, $N_0 > 0$ be constants, $0 \le \theta(t) \le t$, $t \in [0,1]$, $p;\ C_{1-\mu}([0,1]) \to R$ be locally Hölder continuous, and satisfies*

$$\begin{cases} D^{\mu}p(t) + M_0 p(t) + N_0 p(\theta(t)) \ge 0, & t \in (0,1], \\ t^{1-\mu}p(t)|_{t=0} \ge 0. \end{cases}$$

If

$$(M_0 + N_0)\Gamma(1-\mu) < 1,$$

then

$$p(t) \ge 0,\ \forall t \in (0,1].$$

Lemma 2.2. *Let $M > M_1 > 0$, $N > N_1 > 0$ be given, $0 \le \theta(t) \le t$, $t \in [0,1]$. Assume that $v, w \in C_{1-\mu}([0,1])$ satisfy*

$$\begin{cases} D^{\mu}v(t) + Mv(t) + Nv(\theta(t)) - M_1 w(t) - N_1 w(\theta(t)) \ge 0, & t \in (0,1], \\ D^{\mu}w(t) + Mw(t) + Nw(\theta(t)) - M_1 v(t) - N_1 v(\theta(t)) \ge 0, & t \in (0,1], \\ t^{1-\mu}v(t)|_{t=0} \ge 0, \quad t^{1-\mu}w(t)|_{t=0} \ge 0. \end{cases} \tag{2.1}$$

If

$$(M + N + M_1 + N_1)\Gamma(1-\mu) < 1,$$

then

$$v(t) \ge 0, \quad w(t) \ge 0, \quad \forall t \in (0,1].$$

Proof. The proof is similar to the argument of Lemma 2.4 of [15]. In fact, let $p(t) = v(t) + w(t)$, $\forall t \in (0,1]$. Then, by (2.1), we have

$$\begin{aligned} D^{\mu}p(t) &= D^{\mu}(v(t) + w(t)) \\ &\ge -Mv(t) - Nv(\theta(t)) + M_1 w(t) + N_1 w(\theta(t)) \\ &\quad -Mw(t) - Nw(\theta(t)) + M_1 v(t) + N_1 v(\theta(t)) \\ &= -(M - M_1)(v(t) + w(t)) - (N - N_1)(v(\theta(t)) + w(\theta(t))) \\ &= -(M - M_1)p(t) - (N - N_1)p(\theta(t)). \end{aligned}$$