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Perishable inventory system with service interruptions, retrial demands and negative customers

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ABSTRACT

In this paper, we consider a continuous review perishable inventory system wherein demands arrive according to the Poisson process, each demanding exactly one unit of inventory item and the life time of each item is assumed to be exponential. The operating policy is (s, S) policy, i.e., whenever the inventory level drops to s, an order for $Q(=S-s)$ items is placed. The ordered items are received after a random time which is distributed as exponential. The service may be interrupted according to the Poisson process in which case it restarts after an exponentially distributed time. The demands that occur during the server breakdown period or stock-out period may turn out to be ordinary or a negative demand and then they enter into the orbit of infinite size. These orbiting demands send out a signal to compete for their demand which is distributed as exponential. The matrix analytic method is used for the steady state distribution of the model. Various performance measures and cost analysis are shown with numerical results.

1. Introduction

Perishable items are those items, which have a fixed or specified life time after which they are considered unsuitable for utilization. The classical inventory theory did not take into account these perishable items. However, the analysis of perishable inventory systems is important because in real life products like milk, blood, drug, food, vegetables, photographic film and some chemicals do have fixed life times after which they will perish. Weiss [17] introduced the notion of perishable inventories with a continuous review $(0, S)$ policy, Poisson demand and instantaneous supply of ordered items. The author assumed that the items fail after a fixed lifetime. Kalpakam and Arivarignan [4] studied a similar model with (s, S) ordering policy in which items have exponential life times. For recent reviews on perishable inventory systems, see [2,5,9,13].

The delay in service caused by server interruption is a common phenomenon in almost all practical situations. Such interruptions may be caused due to breakdown of a machine, for example, an electronic computer, or the arrival of a high-priority customer that may bring interruption in the service of lower-priority customers. Krishnamoorthy et al. [6] first introduced the concept of service interruptions in an inventory model. They assumed that there is no bound on the number of interruptions that can occur in the middle of a single service and also that an order is instantaneously processed. Later, they extended the above model with positive lead time in [7] and recently in [8] they considered an (s, S) production inventory system with positive service time and the production processes are subject to multiple interruptions.

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The above models consider that the demands that occur during stock-out period or during service interruption period are either backlogged or lost. In the later case, it is assumed that the demands that occur in such situations enter into an orbit of infinite size and retry for their demand after a random time. Artalejo et al. [1] first used the concept of retrial demands in inventory models. They have assumed that an arriving demand at the time of stock-out leaves the service area temporally and repeat its request after some random time. Sivakumar [16] considered a continuous review perishable inventory system with a finite number of homogeneous sources of demands. Periyasamy [14] extended [16] with multiple server vacations.

In all the above models, the authors have considered that the arrival of a customer to the service station should join the system unless it is full. However in some applications an arriving customer may turn out to be a positive or a negative customer. Positive customer joins the system whereas negative customer always removes some positive customers without joining the system. Negative customers can be thought of viruses or commands that delete some transaction in computer network or a database. The concept of queues with negative arrivals was first introduced by Gelenbe [3]. Sivakumar and Arivarignan [15] introduced the concept of negative customers in perishable inventories with finite waiting hall. Manual et al. [10,11] considered a continuous review perishable (s, S) inventory system with two types of customers, ordinary and negative, arrived according to a Markovian arrival process (MAP). In [11], they assumed demands that occur during the stock-out period either enter a pool which has a finite capacity or are lost whereas in [10] such demands are considered as lost.

To the best of our knowledge, there is no literature which takes negative arrivals, retrial demands and service interruptions into consideration in perishable inventory systems. However, in real life there is a chance to form such perishable inventory systems. For example, in a medical store, the drugs can be considered as perishable items. At the time of billing, the service may be interrupted due to a fault in the computer or in the printer. If the drugs required by a customer are not available immediately then he may retry for that drug on the next day (based on two assumptions that the rare drug may not be available nearby or the customer is a regular buyer from the medical shop). On the other hand, the customer may opt for another drug store. Such type of demands are considered as negative customers.

Motivated by the above example in the present paper, we consider an (s, S) policy perishable inventory model with exponential life time for an inventory and service interruption due to server breakdown. Whenever the server is down, it is immediately sent for repair. The broken down time follows Poisson distribution and repair time is exponentially distributed. Primary demands occur according to Poisson distribution. During the service interruption or stock-out period, any arriving primary demands turn to be positive with probability p or negative with probability $q = 1 - p$ and then they enter the orbit of infinite size. These orbiting demands compete for their demands after a random time which is assumed to be exponentially distributed. Using the matrix analytic method, we obtain the steady state distribution of the model. Various performance measures of the model such as expected reorder rate, expected inventory level, expected number of demands in an orbit, etc., are presented. Cost analysis is also carried out using the direct search method.

The rest of the paper is organized as follows. Section 2 and 3 present the description and analysis of the model, respectively. Various performance measures are obtained in Section 4. Section 5 is devoted to the cost analysis which is illustrated by means of numerical examples in Section 6 followed by conclusion in Section 7.

2. Description of the model

Let us consider that the demand arrive according to Poisson process with rate $\lambda(>0)$ and demand only single unit at a time. Life time of each item has exponential distribution with rate $\gamma(>0)$. Inventory is replenished according to (s, S) policy, the replenishment time being exponentially distributed with parameter $\eta(>0)$. The service for a demand may be interrupted due to server breakdown which follows Poisson distribution with parameter δ_1. An interrupted service, after repair, resumes from where it was stopped. The repair time follows exponential distribution with parameter δ_2. We assume that the demands that occur during the stock-out period or during service interruption period may turn out to be an ordinary customer with probability p or a negative customer with probability $q(=1 - p)$ and then they enter the orbit of infinite size. These orbiting customers compete for their demands according to an exponential distribution with parameter $\theta(>0)$. We also assume that no inventory is lost due to service interruption and any order placed in server breakdown state is canceled.

Some notations:

$$c_i = \begin{cases} p\lambda + \eta, & i = 0; \\ \eta + i\gamma, & i = 1, 2, \ldots, s; \\ i\gamma, & i = s + 1, \ldots, S. \end{cases}$$

$$d_i = \begin{cases} p\lambda + \eta + q\lambda, & i = 0; \\ \lambda + \theta + \delta_1 + \eta + i\gamma, & i = 1, 2, \ldots, s; \\ \lambda + \theta + \delta_1 + i\gamma, & i = s + 1, \ldots, S. \end{cases}$$

$$f_i = \lambda + i\gamma, \quad i = 1, 2, \ldots, S.$$

$$h_i = \theta + i\gamma + \lambda, \quad i = 1, \ldots, S.$$

$$g_0 = -(\delta_2 + q\lambda + p\lambda).$$

e : a column vector of appropriate dimension containing all ones.

I_n : an identity matrix of order n.