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Ranking generalized exponential trapezoidal fuzzy numbers based on variance

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ABSTRACT

In this paper, we calculate ranking of exponential trapezoidal fuzzy numbers based on variance. In this method, values are calculated by finding expected values using the probability density function corresponding to the membership functions of the given fuzzy number and provides the correct ordering of exponential trapezoidal fuzzy numbers and also this approach is very simple and easy to apply in the real life problems. For the validation, the results of the approach are compared with different existing approaches.

1. Introduction

In most of the cases in our life, the data obtained for decision making are only approximately known. In 1965, Zadeh [27] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy number as a fuzzy subset of the real line [10]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Ranking fuzzy numbers were first proposed by Jain [11] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Bortolan and Degani [3] reviewed some of these ranking methods [2,6,10,11,14,15] for ranking fuzzy subsets. Chen [4] presented ranking fuzzy numbers with maximizing set and minimizing set. [12] and Wang and Lee [26] also used the centroid concept in developing their ranking index. Chen and Chen [5] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some -levels of trapezoidal fuzzy numbers. Chen and Chen [7] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads and S. K. Barik [3] presented a Probabilistic Quadratic Programming Problems with some fuzzy parameters. Rezvani [14–23] evaluated the system of ranking and properties of fuzzy numbers. Moreover, Rezvani [16] proposed a new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers. In this paper, we calculate ranking of exponential trapezoidal fuzzy numbers based on variance. In this method, values are calculated by finding expected values using the probability density function corresponding to the membership functions of the given fuzzy number and provides the correct ordering of exponential trapezoidal fuzzy numbers and also this approach is very simple and easy to apply in the real life problems. For the validation, the results of the approach are compared with different existing approaches.

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2. Preliminaries

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],
(ii) $\mu_A(x) = 0, -\infty < u \le a$,
(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a, b]$,
(iv) $\mu_A(x) = w, b \le x \le c$,
(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[c, d]$,
(vi) $\mu_A(x) = 0, d \le x < \infty$

Where $0 < w \le 1$ and a, b, c, and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted bye $A = (a, b, c, d; w)_{LR}$. When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by $A = (a, b, c, d)_{LR}$.

However, these fuzzy numbers always have a fix range as $[c, d]$. Here, we define its general form as follows:

$$f_A(x) = \begin{cases} we^{-[(b-x)/(b-a)]} & a \le x \le b, \\ w & b \le x \le c, \\ we^{-[(x-c)/(d-c)]} & c \le x \le d, \end{cases} \tag{1}$$

where $0 < w \le 1$, a, b are real numbers, and c, d are positive real numbers. we denote this type of generalized exponential fuzzy number as $A = (a, b, c, d; w)_E$.

Especially, when $w = 1$, we denote it as $A = (a, b, c, d)_E$. we define the representation of generalized exponential fuzzy number based on the integral value of graded mean h-level as follow. Let the generalized exponential fuzzy number $A = (a, b, c, d)_E$, where $0 < w \le 1$, and c, d are positive real numbers, a, b are real numbers as in formula (2.1). Now, let two monotonic functions be

$$L(x) = we^{-[(b-x)/(b-a)]}, \quad R(x) = we^{-[(x-c)/(d-c)]} \tag{2}$$

3. Proposed approach

As we know that any probability density function with finite support is associated with an expected value, the Mellin transform [24] is used to find this expected value.

Definition 1. The Mellin transform $M_X(t)$ of a probability density function $f(x)$, where x is a positive, is defined as

$$M_X(t) = \int_0^\infty x^{t-1} f(x)\, dx \tag{3}$$

where the integral exists.

Now we find the Mellin transform in terms of expected values. Recall that the expected value of any function $g(X)$ of the random variable X, whose probability density function is $f(x)$, is given by

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx \tag{4}$$

Therefore, it follows that $M_X(t) = E[X^{t-1}] = \int_0^\infty x^{t-1} f(x)\, dx$.

Hence, $E[X^t] = M_X(t+1)$. Thus, the expected value of random variable X is $E[x] = M_X(2)$.

Let $A = (a, b, c, d)_E$ generalized exponential fuzzy number, then the crisp values are calculated by finding expected values using the probability density function corresponding to the membership functions of the given fuzzy number.

Theorem 1. *The probability density function corresponding to exponential trapezoidal fuzzy number $A = (a, b, c, d)_E$ is given as*

$$f_A(x) = C\mu_A(x) \tag{5}$$

where $\mu_A(x)$ is defined as

$$\mu_A(x) = \begin{cases} we^{-[(b-x)/(b-a)]} & a \le x \le b, \\ w & b \le x \le c, \\ we^{-[(x-c)/(d-c)]} & c \le x \le d, \end{cases} \tag{6}$$

Proof. C is calculated as

$$\int_{-\infty}^{\infty} f_A(x)\, dx = 1 \tag{7}$$

that is,

$$\int_{-\infty}^{\infty} C\, \mu_A(x)\, dx = 1 \tag{8}$$