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Oscillatory behavior of second order nonlinear neutral differential equations with distributed deviating arguments

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ABSTRACT

In this article, we shall consider second order nonlinear neutral differential equation of certain type. Some oscillation criteria are established for second-order neutral differential equation of the form

$$[r(t)|z'(t)|^{\gamma-1}z'(t)]' + \int_c^d f(t, x(\sigma(t,\xi)))d\xi = 0,$$

where $z(t) = x(t) + \int_a^b p(t,\xi)x(\tau(t,\xi))d\xi$. An example is given to show the applicability of our results.

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1. Introduction

This article is concerned with the second-order nonlinear neutral differential equation with distributed deviating arguments

$$[r(t)|z'(t)|^{\gamma-1}z'(t)]' + \int_c^d f(t, x(\sigma(t,\xi)))d\xi = 0, \tag{1}$$

where $z(t) = x(t) + \int_a^b p(t,\xi)x(\tau(t,\xi))d\xi$, $\gamma > 0$, $0 \le a < b$, $0 \le c < d$. Throughout this paper it is assumed that

(H_1) $\tau(t,\xi) \in C([t_0,\infty) \times [a,b], (0,\infty))$, $\tau(t,\xi) \le t$ for $\xi \in [a,b]$, $\tau(t,\xi) \to \infty$ as $t \to \infty$,
(H_2) $\sigma(t,\xi) \in C^1([t_0,\infty) \times [c,d], (0,\infty))$, $\sigma(t,\xi)$ is decreasing with respect to ξ, $\sigma(t,\xi) \le t$ for $\xi \in [c,d]$, $\sigma(t,\xi) \to \infty$ as $t \to \infty$, $\sigma_1(t) = \sigma(t,d)$ and $\sigma_1'(t) > 0$,
(H_3) $p(t,\xi) \ge 0$ and $0 \leqslant P(t) = \int_a^b p(t,\xi)d\xi < 1$,
(H_4) $r(t) \in C([t_0,\infty), (0,\infty))$, $\int_{t_0}^{\infty} (\frac{1}{r(t)})^{\frac{1}{\gamma}} dt = \infty$,
(H_5) $f : [t_0,\infty) \times \mathbb{R} \to \mathbb{R}$ is a continuous function such that $uf(t,u) > 0$ for all $u \ne 0$ and there exists a function $q(t,\xi) \in C([t_0,\infty) \times [c,d], [0,\infty))$ such that $|f(t,u)| \ge q(t,\xi)|u^{\gamma}|$.

In recent years, there has been extensive research about the oscillation criteria for second-order delay differential equations. In [1], the second-order delay differential equation of the form

$$(r(t)|u'(t)|^{\alpha-1}u'(t))' + p(t)|u[\tau(t)]|^{\alpha-1}u[\tau(t)] = 0 \tag{2}$$

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was studied. They gave two main results respect to the range of α. Later, (2) was studied by Sun and Meng in [2] and they improved the results in [1]. Furthermore, Dong [3] extended the results in [1] to the second order nonlinear neutral differential equations with deviating arguments of the form

$$(r(t)|z'(t)|^{\alpha-1}z'(t))' + f(t, x[\sigma(t)]) = 0, \quad \alpha > 0,$$

where $z(t) = x(t) + p(t)x(\tau(t))$. Recently, there has been increasing interest in obtaining sufficient conditions for the oscillation of solutions of neutral differential equations with distributed deviating arguments, see [4–6], and the references cited therein. For the books on the subject of neutral differential equations, we refer the reader to [7–11].

The purpose of this article is to give sufficient conditions for the oscillatory behavior of (1) which involves distributed deviating arguments.

The function x is said to be a solution of (1) if the function $z(t)$ and $r(t)|z'(t)|^{\gamma-1}z'(t)$ are continuously differentiable and $x(t)$ satisfies Eq. (1) for $t \geq t_0$. A solution of (1), which is nontrivial for all large t, is called oscillatory if it has no last zero. Otherwise, a solution is called nonoscillatory.

2. Main results

We use the following notations for the simplicity:

$$Q(t) = \int_c^d [1 - P(\sigma(t,\xi))]^\gamma q(t,\xi)d\xi, \quad \tilde{Q}(t) = \int_t^\infty Q(s)ds, \quad \text{and} \quad R(t) = \frac{\gamma\sigma_1'(t)}{r^{\frac{1}{\gamma}}(\sigma_1(t))}.$$

Theorem 2.1. *Assume that*

$$\int_{t_0}^\infty Q(t)dt = \infty. \tag{3}$$

Then, Eq. (1) is oscillatory.

Proof. Let $x(t)$ be a nonoscillatory solution of (1). We assume without loss of generality that $x(t)$ is eventually positive, that is, there exists a $t_0 \geq 0$ such that $x(t) > 0$ for $t \geq t_0$ and therefore there exists a $t_1 \geq t_0$ such that $x(\tau(t,\xi)) > 0$ for $t \geq t_1$ and $\xi \in [a,b]$, $x(\sigma(t,\xi)) > 0$ for $t \geq t_1$ and $\xi \in [c,d]$. If $x(t)$ is an eventually negative solution, it can be proved by the same arguments. From (H_5), we have

$$[r(t)|z'(t)|^{\gamma-1}z'(t)]' = -\int_c^d f(t, x(\sigma(t,\xi)))d\xi \leqslant -\int_c^d q(t,\xi)x^\gamma(\sigma(t,\xi))d\xi \leqslant 0. \tag{4}$$

Hence, $r(t)|z'(t)|^{\gamma-1}z'(t)$ is decreasing. Thus, we have two possible cases for $z'(t)$. (i) $z'(t) < 0$ eventually, (ii) $z'(t) > 0$ eventually.

(i) Assume that $z'(t) < 0$ for $t \geq t_1$. Using decreasing nature of $r(t)|z'(t)|^{\gamma-1}z'(t)$, we obtain

$$r(t)|z'(t)|^{\gamma-1}z'(t) \leqslant r(t_2)|z'(t_2)|^{\gamma-1}z'(t_2), \quad t \geqslant t_2 \geqslant t_1. \tag{5}$$

Dividing both sides of (5) by $r(t)$, integrating from t_2 to t and using (H_4), we obtain

$$z(t) \leqslant z(t_2) - r^{\frac{1}{\gamma}}(t_2)|z'(t_2)| \int_{t_2}^t r^{-\frac{1}{\gamma}}(s)ds \to -\infty \quad \text{as} \quad t \to \infty,$$

which contradicts positive nature of $z(t)$.

(ii) Assume that $z'(t) > 0$ for $t \geq t_1$. Since $z(t) \geq x(t)$ and $z(t)$ is increasing, we have

$$\begin{aligned} z(t) &= x(t) + \int_a^b p(t,\xi)x(\tau(t,\xi))d\xi \\ &\leqslant x(t) + \int_a^b p(t,\xi)z(\tau(t,\xi))d\xi \\ &\leqslant x(t) + \int_a^b p(t,\xi)z(t)d\xi. \end{aligned}$$

Thus, from the last inequality we have

$$(1 - P(t))z(t) \leqslant x(t), \quad t \geqslant t_2^* \geqslant t_1$$

or

$$[(1 - P(\sigma(t,\xi)))z(\sigma(t,\xi))]^\gamma \leqslant x^\gamma(\sigma(t,\xi)), \quad t \geqslant t_3 \geqslant t_2^* \quad \text{and} \quad \xi \in [c,d]. \tag{6}$$

Substituting (6) into (4) and using decreasing nature of $\sigma(t,\xi)$ with respect to ξ, we obtain

$$[r(t)(z'(t))^\gamma]' \leqslant -\int_c^d [1 - P(\sigma(t,\xi))]^\gamma q(t,\xi)z^\gamma(\sigma(t,d))d\xi$$

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