Applied Mathematics and Computation 262 (2015) 204–217

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Periodic solution for stochastic non-autonomous multispecies Lotka–Volterra mutualism type ecosystem ☆

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ARTICLE INFO

Keywords:
Stochastic mutualism model
Periodic solution
Globally attractive
Extinction

ABSTRACT

The paper characterizes some qualitative dynamic properties of a stochastic non-autonomous multi-species mutualism model, with continuous periodic parameters. Using Khasminskii theory of stability with suitable Lyapunov functions, and *M*-Matrices, sufficient conditions are established to guarantee existence of positive periodic solutions to the system. We also provide conditions for the global attractiveness of the latter, or extinction of all species for sufficiently high volatility levels. Results are finally supported by numerical computations.

1. Introduction

Mutualism systems play an important role in population theory, and many researchers have paid their attention to the dynamical behavior of mutualism systems. The classical non-autonomous n-species mutualism model can be represented by

$$dx_i(t) = x_i(t)\left(r_i(t) - a_{ii}(t)x_i(t) + \sum_{j\neq i} a_{ij}(t)x_j(t)\right)dt, \quad i = 1, 2, \ldots, n, \tag{1.1}$$

where $x_i(t)$ is the ith species population density at time t, $r_i(t)$ is the intrinsic growth rate of species x_i at time t, $a_{ii}(t)$ represents the population decay rate in the competition among the ith species at time t, $a_{ij}(t)$ represents the ith species population increase rate in the mutualism among the other species $x_j (i, j = 1, 2, \ldots, n, j \neq i)$ at time t.

There are many researchers who have studied the dynamics of model (1.1), properties such as global attractivity, permanence, non-permanence and global stability of continuous differential mutualism system were extensively studied (see [1–5] and references cited therein). In real world, the environment varies due to the factors such as seasonal effects of weather, food supplies, mating habits, harvesting and so on. So it is reasonable to assume the periodicity of parameters in the systems. There are some nice work on the periodic solutions and almost periodic solutions of continuous and discrete mutualism models with periodic coefficients (see [3,4,6–8] and the references cited therein).

As a matter of fact, only considering the periodicity is not enough, population dynamics is inevitably affected by environmental white noise, which is always present. For this reason, some researchers have paid their attention to the stochastic population models [9–12,13–17]. In this paper, we focus on the case that parameter $r_i(t)$ in model (1.1) is perturbed with white noise,

☆ The work was supported by NSFC of China (no: 11371085), the Ph.D. Programs Foundation of Ministry of China (no. 200918), Tian Yuan Foundation (no. 11426060) and Natural Science Foundation of Changchun Normal University.

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http://dx.doi.org/10.1016/j.amc.2015.04.042
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that is

$$r_i(t) \to r_i(t) + \sigma_i(t)\dot{B}_i(t), \ (i = 1, 2, \ldots, n),$$

then we get the following stochastic system,

$$dx_i(t) = x_i(t)\left[\left(r_i(t) - a_{ii}(t)x_i(t) + \sum_{j \neq i} a_{ij}(t)x_j(t)\right)dt + \sigma_i(t)dB_i(t)\right], \quad i = 1, 2, \ldots, n, \tag{1.2}$$

where $B_i(t)$ is standard one-dimensional Wiener processes (the independence of $Bs_i(t)$ is not necessary in this paper), and $\sigma_i^2(t)$ is the intensity of the white noise at time t.

Liu and Wang [17] obtain the sufficient conditions for stochastic permanence and global attractivity of system (1.2). To our knowledge, there are few work on periodic solution of stochastic differential equations. Periodic solution, however, is an important part of the qualitative analysis of dynamic system. The main purpose of this paper is to obtain sufficient conditions for the existence of positive periodic solution of system (1.2).

The organization of this paper is as follows. We do some preparation work in Section 2. In Section 3, we show that there exists a unique positive global solution under some condition. We obtain the sufficient conditions for the existence of periodic solution in Section 4. Conditions for extinction are given in Section 5. In Section 6, we obtain the conditions for the global attractivity of the periodic solution of the system. We illustrate our results through an example in Section 7. Finally, the conclusion is presented in Section 8.

2. Preliminaries

In this section, we shall prepare some notations, definitions, lemmas which are used in the follows. Throughout this paper, let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets). We denote by R_+^n the positive cone in R^n, that is $R_+^n = \{x \in R^n : x_i > 0 \text{ for all } 1 \leq i \leq n\}$, and denote by $\overline{R}_+^n$ the nonnegative cone in R^n, that is $\overline{R}_+^n = \{x \in R^n : x_i \geq 0 \text{ for all } 1 \leq i \leq n\}$. The norm of x is denoted by $|x| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$. If Q is a matrix, its transpose is denoted by $Q^\top$. If $f(t)$ is a continuous bounded function on $[0, +\infty)$, we define

$$f^u = \sup_{t \in [0,+\infty)} f(t), \qquad f^l = \inf_{t \in [0,+\infty)} f(t).$$

Define

$$\widetilde{A} = \begin{pmatrix} a_{11}^l & -a_{12}^u & \cdots & -a_{1n}^u \\ -a_{21}^u & a_{22}^l & \cdots & -a_{2n}^u \\ \vdots & \vdots & \cdots & \vdots \\ -a_{n1}^u & -a_{n2}^u & \cdots & a_{nn}^l \end{pmatrix},$$

where $a_{ij}^u = \sup_{t \in [0,+\infty)} a_{ij}(t)$, $a_{ij}^l = \inf_{t \in [0,+\infty)} a_{ij}(t)$, and $a_{ij}(t)(i, j = 1, 2, \ldots, n)$ are the coefficients of system (1.2).

We prepare the following assumptions:

(H_1) : $r_i(t), a_{ij}(t), \sigma_i(t)(i, j = 1, 2, \ldots, n)$ are nonnegative continuous θ-periodic functions, and $a_{ii}^l > 0 (i = 1, 2, \ldots, n)$;
(H_2) : $\widetilde{A}$ is a non-singular M-matrix;
(H_3) : $\int_0^\theta r_i(s) - \frac{\sigma_i^2(s)}{2} ds > 0$ for all $i = 1, \ldots, n$;
(H_4) : $r_i^u - \frac{\sigma_i^{2^l}}{2} < 0$ for all $i = 1, \ldots, n$.

Lemma 2.1 ([18]). If $A \in Z^{n \times n}$, where $Z^{n \times n} = \{A = (a_{ij})_{n \times n}) : a_{ij} \leq 0, i \neq j\}$, then the following statements are equivalent:

(1) A is a non-singular M-matrix.
(2) All of the principal minors of A are positive.
(3) *For any $y \gg 0$ in R^n, the linear equation $Ax = y$ has a unique solution $x \gg 0$.*

Lemma 2.2 ([19]). *If A is a non-singular M-matrix, then there is a set of positive numbers $d_1, d_2, \ldots, d_n$ such that the matrix B given by $B = \frac{1}{2}(DA + A^\top D)$ is positive-definite, where $D = diag(d_1, d_2, \ldots, d_n)$.*

Definition 2.1 ([20]). A stochastic process $\xi(t) = \xi(t, \omega)(-\infty < t < \infty)$ is said to be periodic with period θ if for every finite sequence of numbers $t_1, t_2, \ldots, t_n$ the joint distribution of random variables $\xi(t_1 + h), \ldots, \xi(t_n + h)$ is independent of h, where $h = k\theta(k = \pm 1, \pm 2, \ldots)$.