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Synchronization of nonlinear fractional order systems by means of PI$^{r\alpha}$ reduced order observer

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ABSTRACT

This paper deals with the master-slave synchronization scheme for partially known nonlinear fractional order systems, where the unknown dynamics are considered as the master system and we propose the slave system structure which estimates the unknown state variables. Besides it is introduced a new fractional model free reduced order observer inspired on the new concept of Fractional algebraic observability (FAO); we applied the results to a Rössler hyperchaotic fractional order system and Lorenz fractional order system, and by means of some simulations we show the effectiveness of the suggested approach.

1. Introduction

Synchronization of chaotic systems was first initiated by [1]. A variety of observer-based approaches have been proposed for the synchronization of chaotic systems, which include the exponential polynomial observer [2], sliding observer [3], higher order sliding observer [4], fuzzy disturbance observer [5], etc.

In recent years fractional calculus has been used in many fields of engineering and science [6]. It has been shown that the fractional order models of real systems are regularly more adequate than usually used integer order models. There are many systems where the use of fractional differential equations have turned out to be useful, for example systems involved with phenomena such as: viscoelasticity [7], dielectric polarization, electrode–electrolite polarization, stabilization using fractional-order controllers [19] and electromagnetic waves [8].

Synchronization of fractional order chaotic systems was studied by Deng and Li [9] who carried out synchronization in case of the fractional Lu system. Further they have investigated synchronization of the fractional-order Chua system [10].

The main contribution in this paper is the synthesis of a new fractional reduced order observer for the synchronization problem in partially known nonlinear fractional-order systems, we propose a PI$^{r\alpha}$ reduced order observer for estimating the unknown state variables based on Fractional algebraic observability (FAO) property (a system's copy is not necessary). This novel observer presents some advantages, for example, the norm of the estimation error, the time of convergence and the performance of the PI$^{r\alpha}$ reduced order observer can be improved by the correct choice of the gains.

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2. Fractional calculus

Definition 1 (Riemman–Liouville fractional integral)**.** Let $\alpha \in \mathbb{R}^+$ and let f be piecewise continuous on $J' = (0, \infty)$ and integrable on any finite subinterval of $J = [0, \infty)$ (functions of class C). Then for $t > 0$ we call

$$x^{(-\alpha)} = {}_0D_t^{-\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} x(\tau) d\tau, \tag{1}$$

the Riemman–Liouville fractional integral of x of order α. □

There exist several definitions of fractional derivatives of α order [11–14]. However, here we will use the Riemman–Liouville approach.

Definition 2 (Riemman–Liouville fractional derivative [11])**.** Let f be a function of class C and let $\mu > 0$. Let m be the smallest integer that is greater or equal to μ. Then the fractional derivative of f of order μ (if it exists) is defined as

$$D^{\mu} f(t) = D^m \{D^{-v} f(t)\} \quad, \mu > 0, \ t > 0 \tag{2}$$

where $v = m - \mu \geq 0$ □

Note that $v = 0$ implies $D^{\mu} f(t) = D^m f(t)$

Now, we define a sequential operator, as follows.

$$D^{r\alpha} x(t) = \underbrace{[D_t^{\alpha}[D_t^{\alpha} \dots [D_t^{\alpha} x(t)]]]}_{r-times}, \tag{3}$$

i.e., the Riemman–Liouville fractional derivative of order α applied r-times sequentially $r \in \mathbb{N}$, with $D^0 x(t) = x(t)$, we can note that if $r = 1$ then $D^{\alpha} x(t) = x^{(\alpha)}$

2.1. Laplace transform of fractional integrals and fractional derivatives

The Laplace transform is a powerful method in the study of fractional differential–integral equations. In the following paragraphs are introduced some facts about the use of the Laplace transform in fractional calculus [12].

Let f be a function of class C, if f is of exponential order, its fractional integral Laplace transform is given by

$$\mathcal{L}\{D^{-\alpha} f(t)\} = \frac{1}{\Gamma(\alpha)} \mathcal{L}\{f(t)\} = s^{-\alpha} F(s) \quad, \quad \alpha > 0. \tag{4}$$

Let us assume that the Laplace transform of $f(t)$ exists and is denoted by $F(s)$, Thus the following equation holds

$$\mathcal{L}\{D^{\alpha} f(t)\} = s^{\alpha} F(s) - \sum_{k=0}^{m-1} s^{m-k-1} D^{k-m+\alpha} f(0), \tag{5}$$

where $m - 1 < \alpha \leq m$, for $m \in \mathbb{N}$.

We will use these facts in the following problem.

3. Problem statement and main result

We take the initial condition problem for an autonomous fractional order nonlinear system, with $0 < \alpha < 1$:

$$\begin{aligned} x^{(\alpha)} &= f(x), \quad D^{\alpha-1} x(0) = (I_{0+}^{1-\alpha} x)(0+) = x_0 \\ y &= h(x), \end{aligned} \tag{6}$$

where $x \in \Omega \subset \mathbb{R}^n$, $f : \Omega \to \mathbb{R}^n$ is a Lipschitz continuous function (this assures the unique solution [15]), with $x_0 \in \Omega \subset \mathbb{R}^n$, in this case y denotes the output of the system (the measure that we can obtain), $h : \mathbb{R}^n \to \mathbb{R}^q$ is a continuous function and $1 \leq q \leq n$.

Consider the system given in (6), we will separate in two dynamical systems with states $\bar{x} \in \mathbb{R}^p$ which represents the states that we can obtain directly via algebraic relations of the output (known states), and $\eta \in \mathbb{R}^{n-p}$, respectively with $x^T = (\bar{x}^T, \eta^T)$, the first system will describe the known states and the second represents unknown states, then the system (6) can be written as

$$\begin{aligned} \bar{x}^{(\alpha)} &= \bar{f}(\bar{x}, \eta), \\ \eta^{(\alpha)} &= \Delta(\bar{x}, \eta) \\ y &= h(\bar{x}) \end{aligned} \tag{7}$$