Applied Mathematics and Computation 262 (2015) 232–248

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

The direct coupling of local discontinuous Galerkin and natural boundary element method for nonlinear interface problem in $\mathbb{R}^3$

Hongying Huang*

Department of Mathematics, Zhejiang Ocean University, Zhoushan, Zhejiang 316004, China

ARTICLE INFO

MSC:
65N30
65N38

Keywords:
Local discontinuous Galerkin method
Natural boundary reduction
Nonlinear interface problem
Unbounded domain

ABSTRACT

In this article, we use the direct coupling of local discontinuous Galerkin (LDG) and natural boundary element method (NBEM) to solve a class of three-dimensional interface problem, which involves a nonlinear problem in a bounded domain and a Poisson equation in an unbounded domain. A spherical surface as an artificial boundary is introduced. The coupled discrete primal formulation on a bounded domain is obtained. The well-posedness of the primal formulation is verified. The optimal error order with respect to energy norm is given. Numerical examples are presented to demonstrate the optimal convergent rates.

© 2015 Elsevier Inc. All rights reserved.

1. Introduction

The coupling of finite element methods(FEM) and boundary element methods(BEM) is advantageous for problems which involve a finite interior domain(s) embedded in an unbounded exterior domain. The interior domain problem may be nonlinear and complex, whereas the exterior domain problem is relatively simple. The coupling methods are well established in the literature for classical (continuous) finite element and boundary element methods. We refer to [1–5] and the references given therein. However, when the solution in the interior domain is known to be rough, a local discontinuous Galerkin method is certainly more appropriate for its approximation. In particular, LDG methods do not require continuity across the interelement boundaries. It is robust with respect to discontinuous coefficients. It allows the use of different degree polynomials in each element. We refer to [6–13] for recent developments. The coupling of LDG and BEM, as applied to linear exterior boundary value problems on the plane, has been introduced and analyzed in [15]. The corresponding extensions to a class of nonlinear-linear exterior transmission problems were developed recently in [14,16–18]. In [18], the authors enforce the LDG solution space to be continuous on the coupling boundary to avoid mortar variable.

In this article, we will apply the direct coupling of local discontinuous Galerkin and natural boundary element method to solve a class of three dimensional nonlinear-linear exterior interface problems and require approximate functions continuous on the coupling boundary. Natural boundary reduction proposed by Feng and Yu [4] have advantages over the regular boundary reduction methods in many aspects: (1) the coupled bilinear form automatically preserves the symmetry and coerciveness of the original bilinear form, so not only the analysis of the discrete problem is simplified but also the optimal error estimates and the numerical stability are restored; (2) the involved hypersingular boundary integral operator can be transformed into an infinite series via Galerkin variation and the concerned stiff matrix is simple; (3) no new unknowns on the coupled boundary will be introduced and our method only requires solving three equations while the method in [18] needs to solve four equations. Thus it is natural and direct to apply the coupling of FEM or LDG methods with NBE methods under such framework. The disadvantage

* Tel.: +86 13857207022; fax: +86 580 2550029.
E-mail address: huanghy@lsec.cc.ac.cn, hongying@iastate.edu

http://dx.doi.org/10.1016/j.amc.2015.04.036
0096-3003/© 2015 Elsevier Inc. All rights reserved.

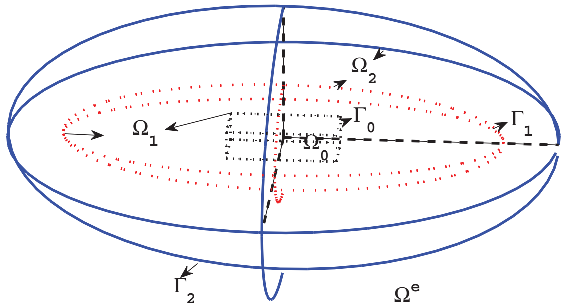

Fig. 1. Domain Ω_0, Ω_1 and Ω_2.

of NBEM is to require a regular shaped boundary, e.g. a circle or ellipse in two dimensions, spherical or ellipsoid surfaces in three dimension. For general problems, we need to introduce an artificial boundary which divides the original unbounded domain into two subregions, a bounded inner region and an unbounded outer region. Thus the original problem is reduced to an equivalent one in the bounded inner region.

In order to introduce the model problem, let Ω_0 be a simply connected and bounded domain in $\mathbb{R}^3$ with polygonal boundary Γ_0. Then, let Ω_1 be an annular and simply connected domain surrounded by Γ_0 and another boundary Γ_1(see Fig. 1). In addition, let $\mathbf{a} : \Omega_1 \times \mathbb{R}^3 \leftarrow \mathbb{R}^3$ be a nonlinear function satisfying the conditions specified in Lipschitz continuous and strongly monotone. Thus, given $f \in L^2(\mathbb{R}^3 \backslash \overline{\Omega}_0)$ with compact support, $g_0 \in H^{1/2}(\Gamma_0)$, $g_1 \in H^{1/2}(\Gamma_1)$, and $g_2 \in L^2(\Gamma_1)$, we consider the nonlinear-linear exterior interface problem:

$$\begin{aligned} -\text{div}\, \mathbf{a}(\mathbf{x}, \nabla u_1) = f \quad \text{in}\ \Omega_1, \quad u_1 = g_0 \quad \text{on}\ \Gamma_0, \\ -\triangle u_2 = f \quad \text{in}\ \mathbb{R}^3 \backslash (\overline{\Omega}_0 \cup \overline{\Omega}_1), \quad u_1 - u_2 = g_1 \quad \text{on}\ \Gamma_1, \\ \mathbf{a}(\mathbf{x}, \nabla u_1) \cdot \mathbf{n} - \nabla u_2 \cdot \mathbf{n} = g_2 \quad \text{on}\ \Gamma_1, \\ u_2(\mathbf{x}) = O\left(\frac{1}{|\mathbf{x}|}\right) \quad \text{as}\ |\mathbf{x}| \to \infty. \end{aligned} \tag{1.1}$$

Here, $\mathbf{n}$ stands for the unit outward normal vector to Γ_1 pointing outside Ω_1.

Next, we introduce a spherical surface $\Gamma_2 = \{\mathbf{x} = (x_1, x_2, x_3) : x_1^2 + x_2^2 + x_3^2 = R^2\}$ such that its interior contains the support of f. Then, let Ω_2 be the annular domain bounded by Γ_1 and Γ_2 and set $\Omega^e = \mathbb{R}^3 \backslash (\overline{\Omega}_0 \cup \overline{\Omega}_1 \cup \overline{\Omega}_2)$ (see Fig. 1). It follows that (1.1) can be equivalently rewritten as the nonlinear boundary value problem in Ω_1:

$$-\text{div}\, \mathbf{a}(\mathbf{x}, \nabla u_1) = f \quad \text{in}\ \Omega_1, \quad u_1 = g_0 \quad \text{on}\ \Gamma_0, \tag{1.2}$$

the Poisson equation in Ω_2:

$$-\triangle u_2 = f \quad \text{in}\ \Omega_2, \tag{1.3}$$

and the Laplace equation in the exterior unbounded region Ω^e:

$$-\triangle u_2 = 0 \quad \text{in}\ \Omega^e, \quad u_2(\mathbf{x}) = O\left(\frac{1}{|\mathbf{x}|}\right) \quad \text{as}\ |\mathbf{x}| \to \infty. \tag{1.4}$$

coupled with the transmission conditions on Γ_1 and Γ_2, respectively,

$$u_1 - u_2 = g_1 \quad \text{and} \quad \mathbf{a}(\mathbf{x}, \nabla u_1) \cdot \mathbf{n} - \nabla u_2 \cdot \mathbf{n} = g_2 \quad \text{on}\ \Gamma_1, \tag{1.5}$$

$$\lim_{\substack{\mathbf{x} \to \mathbf{x}_0 \\ \mathbf{x} \in \Omega^e}} u_2(\mathbf{x}) = \lim_{\substack{\mathbf{x} \to \mathbf{x}_0 \\ \mathbf{x} \in \Omega_2}} u_2(\mathbf{x}) \quad \text{and} \quad \lim_{\substack{\mathbf{x} \to \mathbf{x}_0 \\ \mathbf{x} \in \Omega^e}} \frac{\partial u_2(\mathbf{x})}{\partial \mathbf{n}(x_0)} = \lim_{\substack{\mathbf{x} \to \mathbf{x}_0 \\ \mathbf{x} \in \Omega_2}} \frac{\partial u_2(\mathbf{x})}{\partial \mathbf{n}(x_0)} = \lambda, \tag{1.6}$$

for almost all $\mathbf{x}_0 \in \Gamma_2$, where $\mathbf{n}(x_0)$ denotes the unit outward normal to $\mathbf{x}_0$ pointing exterior to Ω_2.

Throughout this paper, we let $|\cdot|_{s,E}$ and $\|\cdot\|_{s,E}$ denote the seminorm and norm in the space $H^s(E)$, respectively. Let $\mathbf{H}^s(E)$ denote the space $H^s(E) \times H^s(E) \times H^s(E)$. Let $\mathbf{L}^2(E)$ denote the space $L^2(E) \times L^2(E) \times L^2(E)$. $\langle \cdot, \cdot \rangle_{\Gamma_2}$ denotes both the $L^2(\Gamma_2)$ inner product and its extension to the duality pairing of $H^{-s}(\Gamma_2) \times H^s(\Gamma_2)$.

The main purpose of this work is to numerically solve (1.1) by means of the coupled local discontinuous Galerkin(LDG) and Natural boundary element(NBE) approach, which basically consists of applying LDG to (1.2) and (1.3) and NBE method to (1.4). The rest of the paper is organized as follows. In Section 2 we derive the resulting discrete scheme and the primal formulation of the coupled method. This includes the boundary integral equation formulation (i.e., the exact artificial boundary condition on Γ_2) for the exterior problem (1.4), the LDG setting of the interior problem (1.2) and (1.3), and then the coupled LDG-NBE scheme. In Section 3 we define appropriate mesh-dependent norms and prove the boundedness and stability of our discrete scheme. The associated a priori error analysis is provided in Section 4. The good performances of this scheme are illustrated with some simple numerical examples, which also confirm the theoretical rate of convergence of the method.

Download English Version:

https://daneshyari.com/en/article/4626751

Download Persian Version:

https://daneshyari.com/article/4626751

Daneshyari.com