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An approximate solution for a neutral functional–differential equation with proportional delays

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ABSTRACT

In this paper, a novel algorithm based on reproducing kernel theory for neutral functional–differential equation with proportional delays is proposed. The advantages of the presented method are the establishment of complete ε-approximate solution theory and high precision of absolute error. Some examples are given to demonstrate the validity and applicability of the new method and some comparisons are made with the existing results.

1. Introduction

Functional–differential equations with proportional delays are usually referred as pantograph equations or generalized pantograph equations, the name of which was originated by Ockendon and Tayler in connection with the dynamics of a current collection system on an electric locomotive [1]. The pantograph equations occur in a wide variety of many phenomena in applied sciences, such as number theory, electrodynamics, astrophysics, nonlinear dynamical systems, probability theory on algebraic structure, quantum mechanics and cell growth [1–3].

In recent years, pantograph equations have been studied by many authors who have investigated both their analytical and numerical aspects. Here, we briefly review a limited number of them. In [4,15], the multi-pantograph equations were investigated using Runge–Kutta method. The papers [5,6,16,17] studied the numerical solutions of delay differential equations by collocation method. Authors of [7] considered using Chebyshev polynomials method to obtain the numerical solution of the pantograph equation. Sezer et al. [8,9] obtained the approximate solution of the pantograph equation based on the Taylor method. In [10], Alomari et al. used homotopy analysis method to solve a class of delay differential equations. Recently, the authors in [11,12] developed Variational iteration method and Homotopy perturbation method to solve neutral functional–differential equation with proportional delays, respectively. Moreover, Heydari et al. [13] solved pantograph equation with neutral term successfully by the method which consists of expanding the required approximate solution as the elements of Chebyshev cardinal functions.

In this paper, we employ a novel method named optimal residual method to solve the following neutral functional–differential equation with proportional delays,

$$(u(t)+a(t)u(p_m t))^{(m)} = \beta u(t) + \sum_{k=0}^{m-1} b_k(t)u^{(k)}(p_k t) + f(t), \quad 0 \le t \le T \tag{1}$$

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with the initial conditions

$$\sum_{k=0}^{m-1} c_{ik} u^{(k)}(0) = \lambda_i, \quad i = 0, 1, \ldots, m-1, \tag{2}$$

where a and $b_k (k = 0, 1, \ldots, m-1)$ are given analytical functions, and β, p_k, c_{ik}, λ_k denote given constants with $0 < p_k < 1 (k = 0, 1, \ldots, m)$. Differing from the previous methods, here, we will build a complete ε-approximate solution theory based on reproducing kernel theory, and the ε-approximate solution can be easily obtained by solving normal equations.

The rest of the paper is organized as follows. In Section 2, we introduce some mathematical preliminaries about the dense subset of W_2^n. Section 3 is devoted to applying optimal residual method for solving neutral functional-differential equation with proportional delays base on reproducing kernel theory. In Section 4, some numerical results are given to clarify the method, and comparisons are made with methods that were reported in other published works in the literature. Finally, a conclusion is given in Section 5.

2. Preliminaries

In this section, we will prove that $\{1, x, x^2, \ldots, x^{m+1}, \cos kx, \sin kx, k = 1, 2, \cdots\}$ is a completely independent system of W_2^{m+1}.

Lemma 2.1. *The space $W_1 = span\{1, \cos kx, \sin kx, k = 1, 2, 3, \cdots\}$ is a dense subset of $L^2[0, T]$. (see [18]).*

Put

$$W_2^{m+1} \triangleq W_2^{m+1}[0,T] = \{u(x) | u^{(m)}(x) \text{ is an absolutely continuous real} - \text{valued}$$
$$\text{function on } [0,T] \text{ and } u^{(m+1)}(x) \in L^2[0,T]\},$$
$$(u, v) = \sum_{i=0}^{m} u^{(i)}(0) v^{(i)}(0) + \int_0^T u^{(m+1)}(x) v^{(m+1)}(x) \mathrm{d}x, \; u, v \in W_2^{m+1},$$
$$W = span\{1, x, x^2, \ldots, x^{m+1}, \cos kx, \sin kx, k = 1, 2, 3, \ldots\}.$$

Lemma 2.2. *If any $w \in W_1$, then $\int_0^x (x-t)^m w(t) \mathrm{d}t \in W$.*

Proof. Assume that $w = a_0 + \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)$. Letting

$$I_{1,m} = \int_0^x (x-t)^m \mathrm{d}t, \quad I_{2,m} = \int_0^x (x-t)^m \cos kt \mathrm{d}t, \quad I_{3,m} = \int_0^x (x-t)^m \sin kt \mathrm{d}t,$$

then it is enough to show that $I_{1,m}, I_{2,m}, I_{3,m} \in W$. After computing, we obtain

$$I_{1,m} = \frac{x^{m+1}}{m+1}, \quad I_{2,m} = \frac{m}{k} I_{3,m-1}, \quad I_{3,m} = \frac{x^m}{k} - \frac{m}{k} I_{2,m-1}.$$

Obviously, $I_{1,m} \in W$. To prove $I_{2,m}$ and $I_{3,m} \in W$ is to show $I_{2,1}, I_{3,1} \in W$.

In fact, $I_{2,1} = \int_0^x (x-t) \cos kt \mathrm{d}t = -\frac{\cos kx - 1}{k^2} \in W$ and $I_{3,1} = \int_0^x (x-t) \sin kt \mathrm{d}t = -\frac{\sin kx}{k^2} + \frac{x}{k} \in W$. So, our proof is completed. □

Theorem 2.1. *W is a dense subset of W_2^{m+1}.*

Proof. Take any $u \in W_2^{m+1}$ and $\varepsilon > 0$. From $u^{(m+1)} \in L^2[0, T]$ and Lemma 2.1, it follows that there exists a $w \in W_1$ such that

$$\int_0^T (u^{(m+1)}(x) - w(x))^2 \mathrm{d}x = \|u^{(m+1)} - w\|_{L^2}^2 < \varepsilon^2.$$

Put

$$v = \sum_{i=0}^{m} \frac{u^{(i)}(0)}{i!} x^i + \frac{1}{m!} \int_0^x (x-t)^m w(t) \mathrm{d}t \in W.$$

Hence, $u^{(i)}(0) = v^{(i)}(0)$, $i = 0, 1, \ldots, m$ and $v^{(m+1)} = w$. As a result,

$$\|u - v\|_{W_2^{m+1}}^2 = \sum_{i=0}^{m} (u^{(i)}(0) - v^{(i)}(0))^2 + \int_0^T (u^{(m+1)}(x) - v^{(m+1)}(x))^2 \mathrm{d}x$$
$$= \int_0^T (u^{(m+1)}(x) - w)^2 \mathrm{d}x < \varepsilon^2,$$

or

$$\|u - v\|_{W_2^{m+1}} < \varepsilon \text{ with } v \in W.$$

Therefore, W is a dense subset of W_2^{m+1}. □