Applied Mathematics and Computation 260 (2015) 35–47

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Difficulties in detecting chaos in a complex system

Boyan H. Petkov*

Institute of Atmospheric Sciences and Climate (ISAC) of the Italian National Research Council (CNR), Via Gobetti 101, I-40129 Bologna, Italy

ARTICLE INFO

Keywords:
Detection of chaos
Complex chaotic systems
Embedding dimension
Correlation dimension
Lyapunov exponents

ABSTRACT

The sequences, given by a 7D map, are analysed by means of the methods, widely used to detect chaos in the real world in order to test their sensitivity to chaotic features of a non-linear system, determined by comparatively high number of parameters. The same diagnostic approaches are applied to the 3D Lorenz map for comparison. The results show that for some of the sequences yielded from the 7D map, the adopted methods are not able to give a straightforward answer to the question if the system is chaotic as in the 3D case. Since the sequences, subject to the analysis, are not contaminated by noise and are sufficiently long, it could be assumed that such difficulties arise likely due to specific internal features of the more complex system. It is found also that an increase from 0.01 to 0.5 of the sampling step, determining the sequences obtained from the 7D map, masks the chaos in some of them.

1. Introduction

Very often, the decision about chaotic origin of a time series turns out to be a fundamental issue for the study of the processes in the real world [1–5]. Such a judgement is usually made by the assessment of the attractor invariants—Housdorff dimension and Lyapunov exponents [2,3,6,7]. Their correct estimation provides important information about the complexity of the system and its dynamics that helps us to create an adequate model of the phenomenon under study.

The reliability of the methods, developed for detecting chaos in the real systems, is usually tested on time series yielded from well studied maps. A similar test is performed here by applying the adopted methods to the sequences obtained from a 7-dimensional (7D) map, defined by 7 parameters, all connected through nonlinear relationships [8]. Such a map is assumed to imitate a comparatively more complex system and, in order to highlight eventually its specific features, the same test is done with the sequences given by a 3D map. Except for the invariants, the minimal dimension of the space, embedding the reconstructed attractor is also evaluated, since this parameter gives the number of variables composing the system that is also an important characteristic.

To study the sensitivity of the methods that compute the invariants, the researchers usually take one component of a map assuming that the others should represent the same behaviour [1,9–12]. In the present study, these methods are applied to each of the sequences, yielded from both 3D and 7D maps. Such a performance is adopted in order to check if each of the sequences generated by the 7D map is able to depict adequately the system attractor. It is assumed also that the sampling step, determining the sequences under study, is able to influence the topological properties of the reconstructed attractor and hence, the assessment of the parameters chosen to characterise the system. To analyse such an assumption, the evaluations of the corresponding parameters are made by varying the sampling steps of the sequences under study.

* Corresponding author. Tel.: +39 0516399586; fax: +39 0516399652.
E-mail address: b.petkov@isac.cnr.it

http://dx.doi.org/10.1016/j.amc.2015.03.034
0096-3003/

2. Methods of nonlinear time series analysis used in the present study

A starting point for the analysis of a nonlinear system presented by a time series is the reconstruction of the embedding phase space and the attractor. Furthermore, an assessment of the Hausdorff dimension D_0 approximated by estimators allowing easy computation and Lyapunov exponents $\lambda_l (l = 1, 2, \ldots, m)$ is usually performed. Next subsections briefly present some widely used algorithms for estimation of these parameters, which are applied in the present analysis.

2.1. Reconstruction of the system attractor

The reconstruction of the attractor using only one scalar projection [13,14] gave a powerful instrument to study the natural phenomena. For a sequence $(x_i)_{i=1,2,\ldots,N}$ determined by measuring of the variable x at uniquely sampled times $(t_i)_{i=1,2,\ldots,N}$ the Takens's theorem [14] affirms that the m-component vectors constructed as

$$\mathbf{X}_i = (x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(m-1)\tau}), \tag{1}$$

where τ is the so-called time delay, expressed in sampling steps $\Delta t = t_{i+1} - t_i$, determine a manifold that realistically represents the attractor of the system, which generates the time series (x_i). It should be pointed out that there is no a unique optimal choice of the time delay τ [6,15]. In the present study the parameter τ is taken to be the time for which the autocorrelation function of (x_i) drops to e^{-1} or to about 0.37.

2.2. Minimum dimension of the reconstructed embedding space

According to Kennel et al. [16] an acceptable minimum embedding dimension m_{K} of the attractor can be assessed by looking at the behaviour of the nearest neighbour $\mathbf{X}^{(b)}$ of each vector $\mathbf{X}_i$ when the embedding dimension m increases. Assuming Euclidean metric in the phase space the authors found that each $\mathbf{X}^{(b)}$ can be considered as a false nearest neighbour if either of the following two conditions:

$$\frac{|x_{i+m\tau} - x^{(b)}_{i+m\tau}|}{\|\mathbf{X}_i - \mathbf{X}^{(b)}\|^{(\mathrm{E})}_{(m)}} > R_{\mathrm{tol}} \tag{2}$$

and

$$\frac{\|\mathbf{X}_i - \mathbf{X}^{(b)}\|^{(\mathrm{E})}_{(m+1)}}{R_{\mathrm{A}}} > A_{\mathrm{tol}}, \tag{3}$$

is held. The expression $\|\mathbf{X}_i - \mathbf{X}^{(b)}\|^{(\mathrm{E})}_{(m)} = \sqrt{\sum_{k=1}^{m} (x_{i+(k-1)\tau} - x^{(b)}_{i+(k-1)\tau})^2}$ denotes the Euclidean distance between two vectors in m-dimensional embedding spaces and according to Kennel et al. [16] R_{tol} can be considered higher than 10 and $A_{\mathrm{tol}} = 2$. The parameter R_{A} represents the size of the attractor and was taken to be equal to the standard deviation of $(x_i)_{i=1,2,\ldots,N}$. Kennel et al. [16] assumed that the minimum dimension of the embedding space m_{K} for which the false nearest neighbours percentage (FNNP) drops to a value below 1%, allows unfolding of the attractor (see Fig. 4 as an example). Studding the 3D Lorenz system they found also that for a noise free sequence the FNNP remains lower than 1% for $m > m_{\mathrm{K}} = 3$, while a noise contaminated time series shows a different behaviour. For low level of the noise the approach gave $m_{\mathrm{K}} = 4$, whereas for higher level FNNP falls to a value slightly exceeding 1% at $m = m_{\mathrm{K}}$ and plateaus for higher embedding dimension. In the presence of strong noise the length of such a plateau narrows to a few successive values of m and after that FNNP increases. In case of sequence presenting a stochastic process FNNP drops to a comparatively high value (>20%) and after that rapidly increases.

2.3. Correlation dimension of the attractor

Correlation dimension D_2 of the attractor, which is a widely used estimator of the Hausdorff dimension D_0, can be assessed by calculating the correlation integral $C_m(\rho)$:

$$C_m(\rho) = \lim_{N\to\infty} \frac{2}{(N+1-W)(N-W)} \times \sum_{j=W}^{N} \sum_{i=1}^{N-j} \theta\left(\rho - \|\mathbf{X}_i - \mathbf{X}_{i+j}\|^{(\mathrm{Ch})}_{(m)}\right), \tag{4}$$

where $\theta(\xi)$ is the Heaviside function ($\theta(\xi < 0) = 0$ and $\theta(\xi \geq 0) = 1$) and $\|\mathbf{X}_p - \mathbf{X}_q\|^{(\mathrm{Ch})}_{(m)} = \max_{1\leq k\leq m}\{|x_{p+(k-1)\tau} - x_{q+(k-1)\tau}|\}$ is the distance between two vectors in m-dimensional embedding space determined here by the Chebishev metric. The correlation integral $C_m(\rho)$ was defined by Grassberger and Procaccia [17] as Eq. (4) gives it for $W = 1$ and later, Theiler [18,23] proposed the introduction of the cutoff parameter W to avoid a spurious estimate of the correlation dimension resulted from high autocorrelation in the time series under study.

The main point of the Grassberger and Procaccia [17] analysis was the affirmation that for small ρ the correlation integral scales as a power of ρ:

$$C_m(\rho) \sim \rho^{D_m}. \tag{5}$$