Applied Mathematics and Computation 260 (2015) 57–62

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Synchronization of discrete dynamical networks with non-delayed and delayed coupling

Zhaoyan Wu*

College of Mathematics and Information Science, Jiangxi Normal University, Nanchang 330022, China

ARTICLE INFO

Keywords:
Synchronization
Discrete complex network
Non-delayed and delayed coupling

ABSTRACT

In this paper, synchronization of discrete dynamical networks with both non-delayed and delayed couplings are studied. Firstly, a class of complex networks with invariable topology structure are considered. Secondly, a class of complex networks with variable topology structure are discussed. Based on Lyapunov function method and linear matrix inequalities, the sufficient conditions for asymptotically local synchronization of the above two classes of networks are presented respectively. Finally, numerical examples are provided to verify our results.

1. Introduction

Complex dynamical networks have been used to describe many complex systems, in which nodes denote the individual and links denote interaction between them. Such as social network and biological network, social and economic systems, wide-world web and food webs [1–5]. Synchronization is one of the most important collective behaviors of complex networks, which has been studied for decades as a significant and interesting subject [6–17]. In [6], Wang et al. considered synchronization in scale-free dynamical networks. In [13], Liu et al. considered adaptive synchronization in complex dynamical networks for general graph. In [15], Hong et al. studied synchronization of small-world networks and in [16] Fan et al. studied synchronization of scale-free networks.

Time delay exists in many real systems, such as long distance communication and traffic congestion systems, they are nontrivial and cannot be ignored. Therefore networks with retard time were introduced to describe the real networks precisely [18–31]. In [20], Li et al. studied synchronization of complex dynamical networks with time delays. In [22], Yu et al. studied global synchronization of linearly hybrid coupled networks with time-varying delay. In [23], Zheng et al. considered projective synchronization in complex networks with time-varying coupling delay. Especially, in [25–28], authors considered adaptive global synchronization and impulsive synchronization and pinning synchronization in complex networks with non-delayed and delayed coupling respectively.

On the other hand, for better describing the large-scale real systems, discrete-time dynamical networks with or without delays are introduced and investigated [30–34]. In [32], digitally transmitted signals are modelled by discrete-time networks in a dynamical way. In [35], synchronization of discrete-time complex dynamical networks with interval time-varying delays is studied. Motivated by above discussions, in this paper, synchronization of discrete dynamical networks with both non-delayed and delayed couplings will be considered. Firstly, the adjacency matrices of non-delayed coupling and delayed coupling are assumed to be identical, and the sufficient condition for synchronization of the networks can be obtained via linear matrix

* Tel.: +86 791 88120367.
E-mail address: zhywu7981@gmail.com

http://dx.doi.org/10.1016/j.amc.2015.03.044
0096-3003/

inequalities and Lyapunov function method. Secondly, the adjacency matrices of non-delayed coupling and delayed coupling are assumed to be nonidentical, and also be well considered.

This paper is organized as follows. Section 2 introduces the model of discrete dynamical network with both non-delayed and delayed couplings. Section 3 studies the synchronization of these networks and obtains the sufficient condition of synchronization. Section 4 illustrates examples to verify our proposed results. Section 5 concludes the paper.

2. Model and preliminaries

In this section, the discrete dynamical network model with non-delayed and delayed coupling will be introduced. Its state equations are

$$x_i(t+1) = f(x_i(t)) + \varepsilon_1 \sum_{j=1}^{N} a_{ij} g(x_j(t)) + \varepsilon_2 \sum_{j=1}^{N} b_{ij} g(x_j(t-\tau)), \tag{1}$$

where $i = 1, 2, \ldots, N,\ x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ are the state variables of node i and $\varepsilon_1 > 0,\ \varepsilon_2 > 0$ are coupling strength at time t and $t-\tau$ respectively. $A = (a_{ij})_{N\times N}$ and $B = (b_{ij})_{N\times N}$ are the adjacency matrices of non-delayed coupling and delayed coupling respectively. If node i and node j $(i \neq j)$ are connected at time t, then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0\ (i \neq j)$. If node i and node j $(i \neq j)$ are connected at time $t-\tau$, then $b_{ij} = b_{ji} = 1$; otherwise, $b_{ij} = b_{ji} = 0\ (i \neq j)$. The degrees of node i at time t and $t-\tau$ are the number of its connections at each time, denoted by d_{i1} and d_{i2}, which are defined as follows:

$$d_{i1} = -a_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij} = \sum_{j=1, j\neq i}^{N} a_{ji}, \quad i = 1, 2, \ldots, N,$$

$$d_{i2} = -b_{ii} = \sum_{j=1, j\neq i}^{N} b_{ij} = \sum_{j=1, j\neq i}^{N} b_{ji}, \quad i = 1, 2, \ldots, N.$$

The maps $f : R^n \to R^n$ is the dynamics of single node and $g : R^n \to R^n$ is the inner coupling function. The nodes in (1) are said to realize complete (asymptotical) synchronization [6,7] if

$$x_1(t) = x_2(t) = \cdots = x_N(t) \to s(t), \quad as\ t \to \infty,$$

where $s(t) \in R^n$ is a solution of an isolate node, namely $s(t+1) = f(s(t))$, it can be equilibrium or period orbit or chaotic orbit.

3. Main results

In this section, local synchronization of discrete dynamical networks with both non-delayed and delayed couplings will be considered.

Firstly, the adjacency matrices A and B are assumed to be identical i.e., $A = B$. Rewrite the state equations (1) in compact form:

$$X(t+1) = F(X(t)) + \varepsilon_1 G(X(t))A^T + \varepsilon_2 G(X(t-\tau))A^T, \tag{2}$$

where $X(t) = (x_1(t), \ldots, x_N(t)) \in R^{n\times N}$, $F(X) = (f(x_1), \ldots, f(x_N))$, $G(X) = (g(x_1), g(x_2), \ldots, g(x_N))$.

Let $x_i(t) = s(t) + e_i(t)$, $S(t) = (s(t), s(t), \ldots, s(t))$ and $e(t) = (e_1(t), e_2(t), \ldots, e_N(t))$, then get the error equations

$$\begin{aligned} e(t+1) &= F(e(t)+S(t)) - F(S(t)) + \varepsilon_1 (G(e(t)+S(t)) - G(S(t)))A^T \\ &\quad + \varepsilon_2 (G(e(t-\tau)+S(t-\tau)) - G(S(t-\tau)))A^T \\ &= J_1(t)e(t) + \varepsilon_1 W_1(t)e(t)A^T + \varepsilon_2 W_2(t)e(t-\tau)A^T, \end{aligned} \tag{3}$$

where $J_1(t) = Df(s(t)) \in R^{n\times n}$ is the Jacobian of $f(x(t))$ at $s(t)$, $W_1(t) = Dg(s(t))$ and $W_2(t) = Dg(s(t-\tau))$.

Due to A is an irreducible symmetry matrix and there exists a non-singularity matrix Φ such that $A^T\Phi = \Phi\Lambda$, where $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_N)$ and $0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N$ are the eigenvalues of A. Introduce a transformation $\eta(t) = e(t)\Phi$ into (3), one get

$$\eta(t+1) = J_1(t)\eta(t) + \varepsilon_1 W_1(t)\eta(t)\Lambda + \varepsilon_2 W_2(t)\eta(t-\tau)\Lambda, \tag{4}$$

that is,

$$\eta(t+1) = J_1(t)\eta(t) + \varepsilon_1\lambda_i W_1(t)\eta(t) + \varepsilon_2\lambda_i W_2(t)\eta(t-\tau), \quad i = 1, 2, \ldots, N. \tag{5}$$

The eigenvalue $\lambda_1 = 0$ corresponding to the synchronized state of system (2). Then if the following $N-1$ systems of n-dimensional delay equations are asymptotically stable about their zero solutions,

$$\eta(t+1) = J_1(t)\eta(t) + \varepsilon_1\lambda_i W_1(t)\eta(t) + \varepsilon_2\lambda_i W_2(t)\eta(t-\tau), \quad i = 2, 3, \ldots, N, \tag{6}$$

the synchronized state $s(t)$ of system (2) is asymptotically local stable.